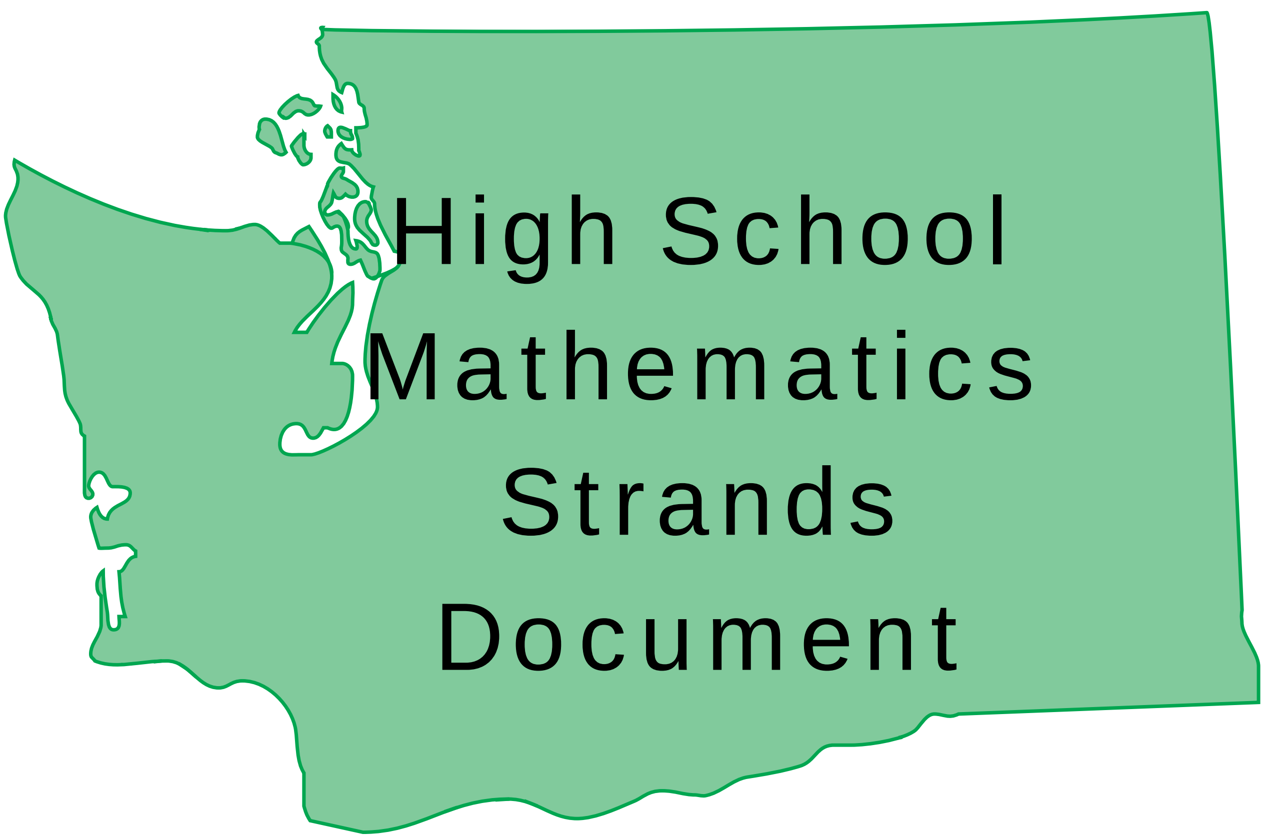
Washington State Mathematics Standards . . . . . .



High School Mathematics Strands Document

(Addendum to K-8 Mathematics Strand Document)
Extracted from August 2008 HS Training Materials CD

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Numbers Strand

In Kindergarten through Grade 2, students learn to use whole numbers to describe sets of objects and locations on the number line. Students in Grades 3-5 extend their understanding of numbers and place value to include fractions and decimals for describing parts of wholes, parts of sets, and locations on the number line. In Grades 6-8, students extend their understanding of positive numbers in fraction and decimal forms and develop an understanding of negative numbers, which along with 0 complete the set of rational numbers**. By the end of high school, students are using real numbers, along with complex numbers, to write numerical and algebraic expressions.**

This is one of six strand documents that accompany the Washington State K–8 Mathematics Standards, tracking the development of important mathematical ideas and skills across grades K–8. Where content of an expectation may address more than one strand, that expectation may appear in more than one strand document.

Algebra 1

A1.2. Core Content: Numbers, expressions, and operations

Performance Expectation	***Explanatory Comments and Examples***

Students are expected to:

A1.2.A Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.

Although a formal definition of real numbers is beyond the scope of Algebra 1, students learn that every point on the number line represents a real number, either rational or irrational, and that every real number has its unique point on the number line. They locate, compare, and order real numbers on the number line.

Real numbers include those written in scientific notation or expressed as fractions, decimals, exponentials, or roots.

Examples:

- Without using a calculator, order the following on the number line:

  $\sqrt{82}$, 3π, 8.9, 9, $\frac{37}{4}$, 9.3×10^0

- A star's color gives an indication of its temperature and age. The chart shows four types of stars and the lowest temperature of each type.

Type	Lowest Temperature (in°F)	Color
A	1.35×10^4	Blue-White
B	2.08×10^4	Blue
G	9.0×10^3	Yellow
P	4.5×10^4	Blue

  List the temperatures in order from lowest to highest.

A1.2.D Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.

Decimal approximations of numbers are sometimes used in applications such as carpentry or engineering, while at other times, these applications may require exact values. Students should understand the difference and know that the appropriate approximation depends upon the necessary degree of precision needed in given situations.

For example, 1.414 is an approximation and not an exact solution to the equation x2 – 2 = 0, but $\sqrt{2}$ is an exact solution to this equation.

Example:

- Using a common engineering formula, an engineering student represented the maximum safe load of a bridge to be $1000(99 - 70\sqrt{2})$ tons. He used 1.41 as the approximation for $\sqrt{2}$ in his calculations. When the bridge was built and tested in a computer simulation to verify its maximum weight-bearing load, it collapsed! The student had estimated the bridge would hold ten times the weight that was applied to it when it collapsed.

- Calculate the weight that the student thought the bridge could bear using 1.41 as the estimate for $\sqrt{2}$.
- Calculate other weight values using estimates of $\sqrt{2}$ that have more decimal places. What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.

Algebra 2

A2.2. Core Content: Numbers, expressions, and operations

Performance Expectation	***Explanatory Comments and Examples***
Students are expected to:	
A2.2.A Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.	Example: Within which number system(s) can each of the following be solved? Explain how you know. • $3x + 2 = 5$ • $x^2 = 1$ • $x^2 = \frac{1}{4}$ • $x^2 = 2$ • $x^2 = -2$ • $\frac{x}{7} = \pi$

Mathematics 1

M1.6. Core Content: Numbers, expressions, and operations

Performance Expectation	***Explanatory Comments and Examples***
Students are expected to:	
M1.6.A Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.	Although a formal definition of real numbers is beyond the scope of Mathematics 1, students learn that every point on the number line represents a real number, either rational or irrational, and that every real number has its unique point on the number line. They locate, compare, and order real numbers on the number line.

Real numbers include those written in scientific notation or expressed as fractions, decimals, exponentials, or roots.

Examples:

- Without using a calculator, order the following on the number line:
  $\sqrt{82}$, 3π, 8.9, 9, $\frac{37}{4}$, 9.3×10^0
- A star's color gives an indication of its temperature and age. The chart shows four types of stars and the lowest temperature of each type.

Type	Lowest Temperature (in °F)	Color
A	1.35×10^4	Blue-White
B	2.08×10^4	Blue
G	9.0×10^3	Yellow
P	4.5×10^4	Blue

List the temperatures in order from lowest to highest.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

M1.6.B Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.

Decimal approximations of numbers are sometimes used in applications such as carpentry or engineering, while at other times, these applications may require exact values. Students should understand the difference and know that the appropriate approximation depends upon the necessary degree of precision needed in given situations.

For example, 1.414 is an approximation and not an exact solution to the equation $x^2 - 2 = 0$, but $\sqrt{2}$ is an exact solution to this equation.

Example:

- Using a common engineering formula, an engineering student represented the maximum safe load of a bridge to be $1000(99 - 70\sqrt{2})$ tons. He used 1.41 as the approximation for $\sqrt{2}$ in his calculations. When the bridge was built and tested in a computer simulation to verify its maximum weight-bearing load, it collapsed! The student had estimated the bridge would hold ten times the weight that was applied to it when it collapsed.
  - — Calculate the weight that the student thought the bridge could bear using 1.41 as the estimate for $\sqrt{2}$.
  - — Calculate other weight values using estimates of $\sqrt{2}$ that have more decimal places. What might be a reasonable degree of precision required to know how much weight the bridge can handle safely? Justify your answer.

Mathematics 3

M3.6. Core Content: Algebraic properties

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	

M3.6.A Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.

Example: Within which number system(s) can each of the following be solved? Explain how you know.

- $3x + 2 = 5$
- $x^2 = 1$
- $x^2 = \frac{1}{4}$
- $x^2 = 2$
- $x^2 = -2$
- $\frac{x}{7} = \pi$

Operations Strand

As they use their knowledge of whole numbers to describe situations that involve joining, separating, and comparing, students in Kindergarten through Grade 2 learn to add and subtract whole numbers and use these operations to solve problems. In Grades 3–5, students learn the meaning of multiplication and division to describe situations that involve joining equal groups and separating sets into equal-sized groups. Students learn to multiply and divide whole numbers, and they select from addition, subtraction, multiplication, and division the appropriate operations needed to solve problems. Also in Grades 3–5, students use their understanding of place value to begin to add and subtract decimals. At the same time, students use what they have learned about the meaning of fractions and equivalent fractions to begin to add and subtract fractions. In Grades 6–8, students apply their understanding of fractions, decimals, and properties of the operations to extend their computational fluency to add, subtract, multiply, and divide rational numbers. Students build on their understanding of multiplication and division to develop important concepts about ratios, rates, and proportional relationships. **By the end of high school, students are evaluating expressions and solving equations with real numbers.**

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Algebra 1

A1.2. Core Content: *Numbers, expressions, and operations*

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
A1.2.C Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.	Examples: • $2^{-3} = \frac{1}{2^3}$ • $\frac{2^{-2}3^{2}5}{2^{2}3^{-3}5^{2}} = \frac{3^5}{2^4 5}$ • $\frac{a^{-2}b^{2}c}{a^{2}b^{-3}c^{2}} = \frac{b^5}{a^4 c}$ • $\sqrt{8} = \sqrt{2 \bullet 2 \bullet 2} = 2\sqrt{2}$ • $\sqrt[3]{a \bullet b} = \sqrt[3]{a} \bullet \sqrt[3]{b}$

Algebra 2

A2.2. Core Content: Numbers, expressions, and operations

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
A2.2.B Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.	Examples: • Convert the following from a radical to exponential form or visa versa. • $24^{\frac{1}{3}}$ • $\sqrt[5]{16}$ • $\sqrt{x^2 + 1}$ • $\frac{x^2}{\sqrt{x}}$ • Evaluate $x^{-3/2}$ for $x = 27$

A2.4. Core Content: Exponential and logarithmic functions and equations

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
A2.4.A Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.	Examples: • Given f(x) = 4x, write an equation for the inverse of this function. Graph the functions on the same coordinate grid. • Find f(-3). • Evaluate the inverse function at 7. • Derive the formulas: • $\log_b a \cdot \log_a b = 1$ • $\log_a N = \log_b N \cdot \log_a b$ • Find the exact value of x in: • $\log_x 16 = \frac{4}{3}$ • $\log_3 81 = x$ • Solve for y in terms of x: • $\log_a \frac{y}{x} = x$ • 100 = x · 10y

Mathematics 1

M1.7. Additional Key Content:

Performance Expectation	*Explanatory Comments and Examples*
Students are expected to:	
M1.7.C Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.	Examples: • $2^{-3} = \frac{1}{2^3}$ • $\frac{2^{-2}3^2 5}{2^2 3^{-3} 5^2} = \frac{3^5}{2^4 5}$ • $\frac{a^{-2}b^2 c}{a^2 b^{-3} c^2} = \frac{b^5}{a^4 c}$ • $\sqrt{8} = \sqrt{2 \bullet 2 \bullet 2} = 2\sqrt{2}$ • $\sqrt[3]{a \bullet b} = \sqrt[3]{a} \bullet \sqrt[3]{b}$

Mathematics 3

M3.3. Core Content: *Functions and modeling*

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M3.3.A Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.	Examples: • Given $f(x) = 4x$, write an equation for the inverse of this function. Graph the functions on the same coordinate grid. • Find $f(-3)$. • Evaluate the inverse function at 7. • $\log_b a \cdot \log_a b = 1$ • $\log_a N = \log_b N \cdot \log_a b$ • Find the exact value of x in: • $\log_x 16 = \frac{4}{3}$ • $\log_3 81 = x$ • Solve for y in terms of x: • $\log_a \frac{y}{x} = x$ • $100 = x \cdot 10y$

M3.6. Core Content: *Algebraic properties*

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M3.6.B Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.	Examples: • Convert the following from a radical to exponential form or visa versa. • $24^{\frac{1}{3}}$ • $\sqrt[5]{16}$ • $\sqrt{x^2+1}$ • $\frac{x^2}{\sqrt{x}}$ • Evaluate $x^{-3/2}$ for $x = 27$

Algebra Strand

In grades K–5, students prepare for algebra by learning about the properties of arithmetic and by describing rules for patterns. They begin to work with number sentences as they build an understanding of equality. As a transition from arithmetic to more formal algebra, students in grades 6–8 use numbers and variables in mathematical expressions to describe situations and use their understanding of numbers and operations to solve linear equations and deal with formulas. **By the end of high school, students are working with many different types of functions, equations, and inequalities, including those involving quadratics and exponents.**

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Algebra 1

A1.1. Core Content: Solving problems

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
A1.1.A Select and justify functions and equations to model and solve problems.	Students can analyze the rate of change of a function represented with a table or graph to determine if the function is linear. Students also analyze common ratios to determine if the function is exponential.

After selecting a function to model a situation, students describe appropriate domain restrictions. They use the function to solve the problem and interpret the solution in the context of the original situation.

Examples:

- A cup is 6 cm tall, including a 1.1 cm lip. Find a function that represents the height of a stack of cups in terms of the number of cups in the stack. Find a function that represents the number of cups in a stack of a given height.
- For the month of July, Michelle will be dog-sitting for her very wealthy, but eccentric, neighbor, Mrs. Buffett. Mrs. Buffett offers Michelle two different salary plans:

  Plan 1: $100 per day for the 31 days of the month.

  Plan 2: $1 for July 1, $2 for July 2, $4 for July 3, and so on, with the daily rate doubling each day.

  a. Write functions that model the amount of money Michelle will earn each day on Plan 1 and Plan 2. Justify the functions you wrote.

  b. State an appropriate domain for each of the models based on the context.

  c. Which plan should Michelle choose to maximize her earnings? Justify your recommendation mathematically.

  d. Extension: Write an algebraic function for the cumulative pay for each plan based on the number of days worked.

A1.1.B Solve problems that can be represented by linear functions, equations, and inequalities.

It is mathematically important to represent a word problem as an equation. Students must analyze the situation and find a way to represent it mathematically. After solving the equation, students think about the solution in terms of the original problem. Examples:

- The assistant pizza maker makes 6 pizzas an hour. The master pizza maker makes 10 pizzas an hour but starts baking two hours later than his assistant. Together, they must make 92 pizzas. How many hours from when the assistant starts baking will it take?
- What is a general equation, in function form, that could be used to determine the number of pizzas that can be made in two or more hours?
- A swimming pool holds 375,000 liters of water. Two large hoses are used to fill the pool. The first hose fills at the rate of 1,500 liters per hour and the second hose fills at the rate of 2,000 liters per hour. How many hours does it take to fill the pool completely?

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***

A1.1.C Solve problems that can be represented by a system of two linear equations or inequalities.

Examples:

- An airplane flies from Baltimore to Seattle (assume a distance of 2,400 miles) in 7 hours, but the return flight takes only $4\frac{1}{2}$ hours. The air speed of the plane is the same in both directions. How many miles per hour does the plane fly with respect to the wind? What is the wind speed in miles per hour?
- A coffee shop employee has one cup of 85% milk (the rest is chocolate) and another cup of 60% milk (the rest is chocolate). He wants to make one cup of 70% milk How much of the 85% milk and 60% milk should he mix together to make the 70% milk?
- Two plumbing companies charge different rates for their service. Clyde's Plumbing Company charges a $75-per-visit fee that includes one hour of labor plus $45 dollars per hour after the first hour. We-Unclog-It Plumbers charges a $100-per-visit fee that includes one hour of labor plus $40 per hour after the first hour. For how many hours of plumbing work would Clyde's be less expensive than We-Unclog-It?
  Note: Although this context is discrete, students can model it with continuous linear functions.

A1.1.D Solve problems that can be represented by quadratic functions and equations.

Examples:

- Find the solutions to the simultaneous equations $y = x + 2$ and $y = x^2$.
- If you throw a ball straight up (with initial height of 4 feet) at 10 feet per second, how long will it take to fall back to the starting point? The function $h(t) = -16t^2 + v_0t + h_0$ describes the height, h in feet, of an object after t seconds, with initial velocity v_0 and initial height h_0.
- Joe owns a small plot of land 20 feet by 30 feet. He wants to double the area by increasing both the length and the width, keeping the dimensions in the same proportion as the original. What will be the new length and width?
- What two consecutive numbers, when multiplied together, give the first number plus 16? Write the equation that represents the situation.

A1.1.E Solve problems that can be represented by exponential functions and equations.

Students approximate solutions with graphs or tables, check solutions numerically, and when possible, solve problems exactly.

Examples:

- E. coli bacteria reproduce by a simple process called binary fission—each cell increases in size and divides into two cells. In the laboratory, E. coli bacteria divide approximately every 15 minutes. A new E. coli culture is started with 1 cell.
  a. Find a function that models the E. coli population size at the end of each 15-minute interval. Justify the function you found.
  b. State an appropriate domain for the model based on the context.
  c. After what 15-minute interval will you have at least 500 bacteria?
- Estimate the solution to $2^x = 16,384$

A1.2. Core Content: Numbers, expressions, and operations

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A1.2.B Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.	Students learn to use letters as variables and in other ways that increase in sophistication throughout high school. For example, students learn that letters can be used: To represent fixed and temporarily unknown values in equations, such as $3x + 2 = 5$; To express identities, such as $x + x = 2x$ for all x; As attributes in formulas, such as $A = lw$; As constants such as a, b, and c in the equation $y = ax2 + bx + c$; As parameters in equations, such as the m and b for the family of functions defined by $y = mx + b$; To represent varying quantities, such as x in $f(x) = 5x$; To represent functions, such as f in $f(x) = 5x$; and To represent specific numbers, such as π. Expressions include those involving polynomials, radicals, absolute values, and integer exponents. Examples: • For what values of a and n, where n is an integer greater than 0, is an always negative? • For what values of a is $\frac{1}{a}$ an integer?
A1.2.E Use algebraic properties to factor and combine like terms in polynomials.	Algebraic properties include the commutative, associative, and distributive properties. Factoring includes: • Factoring a monomial from a polynomial, such as $4x^2 + 6x = 2x(2x + 3)$; • Factoring the difference of two squares, such as $36x^2 – 25y^2 = (6x + 5y)(6x – 5y)$ and $x^4 – y^4 = (x + y)(x – y)(x^2 + y^2)$; • Factoring perfect square trinomials, such as $x^2 + 6xy + 9y^2 = (x + 3y)^2$; • Factoring quadratic trinomials such as $x^2 + 5x + 4 = (x + 4)(x + 1)$; and • Factoring trinomials that can be expressed as the product of a constant and a trinomial, such as $0.5x^2 – 2.5x – 7 = 0.5(x + 2)(x – 7)$.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.2.F Add, subtract, multiply, and divide polynomials.	Write algebraic expressions in equivalent forms using algebraic properties to perform the four arithmetic operations with polynomials. Students should recognize that expressions are essentially sums, products, differences, or quotients. For example, the sum $2x^2 + 4x$ can be written as a product, $2x(x + 2)$. Examples: • $(3x^2 – 4x + 5) + (-x^2 + x – 4) + (2x^2 + 2x + 1)$ • $(2x^2 – 4) – (x^2 + 3x – 3)$ • $\frac{2x^2}{9} \bullet \frac{6}{2x^4}$

A1.3. Core Content: Characteristics and behaviors of functions

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.3.A Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.	Functions studied in Algebra 1 include linear, quadratic, exponential, and those defined piecewise (including step functions and those that contain the absolute value of an expression). Given a problem situation, students should describe further restrictions on the domain of a function that are appropriate for the problem context. Examples: • Which of the following are functions? Explain why or why not. The age in years of each student in your math class and each student's shoe size. The number of degrees a person rotates a spigot and the volume of water that comes out of the spigot. • A function $f(n) = 60n$ is used to model the distance in miles traveled by a car traveling 60 miles per hour in n hours. Identify the domain and range of this function. What restrictions on the domain of this function should be considered for the model to correctly reflect the situation? • What is the domain of $f(x) = \sqrt{5 – x}$?

- Which of the following equations, inequalities, or graphs determine *y* as a function of *x*?

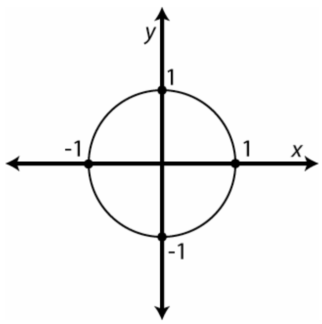
$y = 2$

$x = 3$

$y = |x|$

$$y = \begin{cases} x + 3, & x \le 1 \\ x - 2, & x > 1 \end{cases}$$

$x^2 + y^2 = 1$



Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A1.3.B Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.	This expectation applies each time a new class (family) of functions is encountered. In Algebra 1, students should be introduced to a variety of additional functions that include expressions such as x^3, $\sqrt{x}$, $\frac{1}{x}$, and absolute values. They will study these functions in depth in subsequent courses. Students should know that f(*x*) = $\frac{a}{x}$ represents an inverse variation. Students begin to describe the graph of a function from its symbolic expression, and use key characteristics of the graph of a function to infer properties of the related symbolic expression. Translating among these various representations of functions is an important way to demonstrate conceptual understanding of functions. Students learn that each representation has particular advantages and limitations. For example, a graph shows the shape of a function, but not exact values. They also learn that a table of values may not uniquely determine a single function without some specification of the nature of that function (e.g., it is quadratic).
A1.3.C Evaluate *f*(*x*) at *a* (i.e., *f*(*a*)) and solve for *x* in the equation *f*(*x*) = *b*.	Functions may be described and evaluated with symbolic expressions, tables, graphs, or verbal descriptions. Students should distinguish between solving for *f*(*x*) and evaluating a function at *x*.

Example:

- Roses-R-Red sells its roses for $0.75 per stem and charges a $20 delivery fee per order.

  What is the cost of having 10 roses delivered?

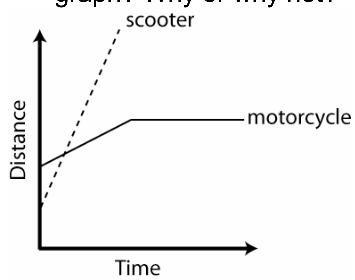
  How many roses can you have delivered for $65?

A1.4. Core Content: Linear functions, equations, and inequalities

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
A1.4.A Write and solve linear equations and inequalities in one variable.	This expectation includes the use of absolute values in the equations and inequalities. Examples: • Write an absolute value equation or inequality for all the numbers 2 units from 7, and all the numbers that are more than *b* units from *a*. • Solve $\|x - 6\| \leq 4$ and locate the solution on the number line. • Write an equation or inequality that has no real solutions; infinite numbers of real solutions; and exactly one real solution. • Solve for x in $2(x - 3) + 4x = 15 + 2x$. • Solve $8.5 < 3x + 2 \leq 9.7$ and locate the solution on the number line.
A1.4.B Write and graph an equation for a line given the slope and the *y*-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.	Linear equations may be written in slope-intercept, point-slope, and standard form. Examples: • Find an equation for a line with *y*-intercept equal to 2 and slope equal to 3. • Find an equation for a line with a slope of 2 that goes through the point (1, 1). • Find an equation for a line that goes through the points (-3, 5) and (6, -2). • For each of the following, use only the equation (without sketching the graph) to describe the graph. $y = 2x + 3$ $y - 7 = 2(x - 2)$ • Write the equation $3x + 2y = 5$ in slope intercept form. Write the equation $y - 1 = 2(x - 2)$ in standard form.

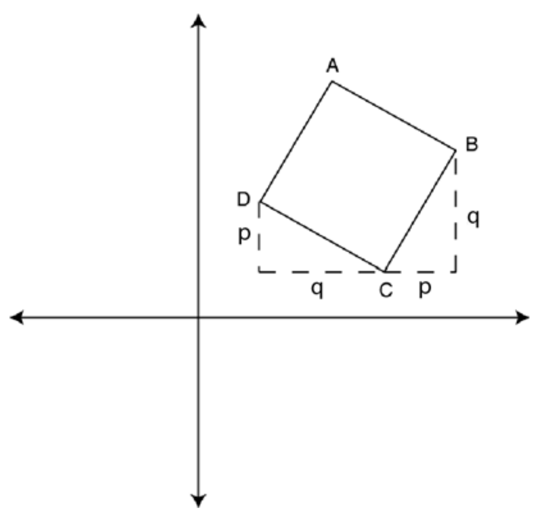
Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.4.C Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.	Examples:

- The graph shows the relationship between time and distance from a gas station for a motorcycle and a scooter. What can be said about the relative speed of the motorcycle and scooter that matches the information in the graph? What can be said about the intersection of the graphs of the scooter and the motorcycle? Is it possible to tell which vehicle is further from the gas station at the initial starting point represented in the graph? At the end of the time represented in the graph? Why or why not?



- A 1,500-gallon tank contains 200 gallons of water. Water begins to run into the tank at the rate of 75 gallons per hour. When will the tank be full? Find a linear function that models this situation, draw a graph, and create a table of data points. Once you have answered the question and completed the tasks, explain your reasoning. Interpret the slope and *y*-intercept of the function in the context of the situation.

- Given that the figure below is a square, find the slope of the perpendicular sides AB and BC. Describe the relationship between the two slopes.



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Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***

A1.4.D Write and solve systems of two linear equations and inequalities in two variables.

Students solve both symbolic and word problems, understanding that the solution to a problem is given by the coordinates of the intersection of the two lines when the lines are graphed in the same coordinate plane.

Examples:

- Solve the following simultaneous linear equations algebraically:

  $-2x + y = 2$

  $x + y = -1$

- Graph the above two linear equations on the same coordinate plane and use the graph to verify the algebraic solution.
- An academic team is going to a state mathematics competition. There are 30 people going on the trip. There are 5 people who can drive and 2 types of vehicles, vans and cars. A van seats 8 people, and a car seats 4 people, including drivers. How many vans and cars does the team need for the trip? Explain your reasoning.

Let v = number of vans and c = number of cars.

$$v + c \leq 5$$
$$8v + 4c \geq 30$$

A1.4.E Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.

In the case of a linear function $y = f(x)$, expressed in slope-intercept form ($y = mx + b$), m and b are parameters. Students should know that $f(x) = kx$ represents a direct variation (proportional relationship).

Examples:

- Graph a function of the form $f(x) = kx$, describe the effect that changes on k have on the graph and on $f(x)$, and answer questions that arise in proportional situations.
- A gas station's 10,000-gallon underground storage tank contains 1,000 gallons of gasoline. Tanker trucks pump gasoline into the tank at a rate of 400 gallons per minute. How long will it take to fill the tank? Find a function that represents this situation and then graph the function. If the flow rate increases from 400 to 500 gallons per minute, how will the graph of the function change? If the initial amount of gasoline in the tank changes from 1,000 to 2,000 gallons, how will the graph of the function change?

Compare and contrast the functions $y = 3|x|$ and $y = \text{Š}\frac{1}{3}|x|$.

A1.5. Core Content: Quadratic functions and equations

Performance Expectation Students are expected to:	Explanatory Comments and Examples

A1.5.A Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.

Example:

- Kendre and Tyra built a tennis ball cannon that launches tennis balls straight up in the air at an initial velocity of 50 feet per second. The mouth of the cannon is 2 feet off the ground. The function $h(t) = -16t^2 + 50t + 2$ describes the height, h, in feet, of the ball t seconds after the launch.

  Make a table from the function. Then use the table to sketch a graph of the height of the tennis ball as a function of time into the launch. Give a verbal description of the graph. How high was the ball after 1 second? When does it reach this height again?

A1.5.B Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the x-intercepts as solutions to a quadratic equation.

Note that in Algebra 1, the parameter b in the term bx in the quadratic form $ax^2 + bx + c$ is not often used to provide useful information about the characteristics of the graph.

Parameters considered most useful are:

- a and c in $f(x) = ax^2 + c$
- a, h, and k in $f(x) = a(x – h)^2 + k$, and
- a, r, and s in $f(x) = a(x – r)(x – s)$

Example:

- A particular quadratic function can be expressed in the following two ways:

  $f(x) = -(x – 3)^2 + 1$
  $f(x) = -(x – 2)(x – 4)$

  — What information about the graph can be directly inferred from each of these forms? Explain your reasoning.

  — Sketch the graph of this function, showing the roots.

A1.5.C Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where a, b, c, and d are integers.

Students learn to efficiently solve quadratic equations by recognizing and using the simplest factoring methods, including recognizing special quadratics as squares and differences of squares.

Examples:

- $2x^2 + x – 3 = 0$; $(x – 1)(2x + 3) = 0$; $x = 1, -\frac{3}{2}$
- $4x^2 + 6x = 0$; $2x(2x + 3) = 0$; $x = 0$, Š$\frac{3}{2}$
- $36x^2 – 25 = 0$; $(6x + 5)(6x – 5) = 0$; $x = \pm\frac{5}{6}$
- $x^2 + 6x + 9 = 0$; $(x + 3)^2 = 0$; $x = –3$

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
A1.5.D Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.	Students solve those equations that are not easily factored by completing the square and by using the quadratic formula. Completing the square should also be used to derive the quadratic formula. Students learn how to determine if there are two, one, or no real solutions. Examples: • Complete the square to solve $x^2 + 4x = 13$. $x^2 + 4x - 13 = 0$ $x^2 + 4x + 4 = 17$ $(x + 2)^2 = 17$ $x + 2 = \pm\sqrt{17}$ $x = -2 \pm\sqrt{17}$ $x \approx 2.12, -6.12$ • Use the quadratic formula to solve $4x^2 - 2x = 5$. $x \approx 1.40, -0.90$

A1.7. Additional Key Content:

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
A1.7.A Sketch the graph for an exponential function of the form $y = ab^n$ where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.	Examples: • Sketch the graph of $y = 2^n$ by hand. • You have won a door prize and are given a choice between two options: $150 invested for 10 years at 4% compounded annually. $200 invested for 10 years at 3% compounded annually. How much is each worth at the end of each year of the investment periods? Are the two investments ever equal in value? Which will you choose?
A1.7.B Find and approximate solutions to exponential equations.	Students can approximate solutions using graphs or tables with and without technology.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.7.C Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.	Examples: • Write a recursive formula for the arithmetic sequence 5, 9, 13, 17, . . . What is the slope of the line that contains the points associated with these values and their position in the sequence? How is the slope of the line related to the sequence? • Given that $u(0) = 3$ and $u(n + 1) = u(n) + 7$ when n is a positive integer, a. find $u(5)$; b. find n so that $u(n) = 361$; and c. find a formula for $u(n)$. • Write a recursive formula for the geometric sequence 5, 10, 20, 40, . . . and determine the 100th term. • Given that $u(0) = 2$ and $u(n + 1) = 3u(n)$, a. find $u(4)$, and b. find a formula for $u(n)$.
A1.7.D Solve an equation involving several variables by expressing one variable in terms of the others.	Examples: • Solve $A = p + prt$ for p. Solve $V = \pi r^2 h$ for h or for r.

Algebra 2

A2.1. Core Content: Solving problems

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***

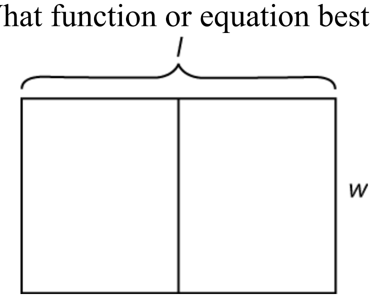
A2.1.A Select and justify functions and equations to model and solve problems.

Examples:

- A manufacturer wants to design a cylindrical soda can that will hold 500 milliliters (mL) of soda. The manufacturer's research has determined that an optimal can height is between 10 and 15 centimeters. Find a function for the radius in terms of the height, and use it to find the possible range of radius measurements in centimeters. Explain your reasoning.
- Dawson wants to make a horse corral by creating a rectangle that is divided into 2 parts, similar to the following diagram. He has a 1200-foot roll of fencing to do the job.

  What are the dimensions of the enclosure with the largest total area?

  What function or equation best models this situation?



A2.1.B Solve problems that can be represented by systems of equations and inequalities.

Examples:

- Mr. Smith uses the following formula to calculate students' final grades in his Algebra 1I class: $0.4E + 0.6T = C$, where E represents the score on the final exam, and T represents the average score of all tests given during the grading period. All tests and the final exam are worth a maximum of 100 points. The minimum passing score on tests, the final exam, and the course is 60.
- Determine the inequalities that describe the following situation and sketch a system of graphs to illustrate it. When necessary, round scores to the nearest tenth.
- Is it possible for a student to have a failing test score average (i.e., $T < 60$ points) and still pass the course?
- If you answered "yes," what is the minimum test score average a student can have and still pass the course? What final exam score is needed to pass the course with a minimum test score average?
- A student has a particular test score average. How can (s)he figure out the minimum final exam score needed to pass the course?
- Data derived from an experiment seems to be parabolic when plotted on a coordinate grid. Three observed data points are (2, 10), (3, 8), and (4, 4). Write a quadratic equation that passes through the points.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.	In addition to solving area and velocity problems by factoring and applying the quadratic formula to the quadratic equation, students use the vertex form of the equation to solve problems about maximums, minimums, and symmetry. Examples: • The Gateway Arch in St. Louis has a special shape called a catenary, which looks a lot like a parabola. It has a base width of 600 feet and is 630 feet high. Which is taller, this catenary arch or a parabolic arch that has the same base width but has a height of 450 feet at a point 150 feet from one of the pillars? What is the height of the parabolic arch? • Fireworks are launched upward from the ground with an initial velocity of 160 feet per second. The formula for vertical motion is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second, v represents the initial velocity, and s represents the initial height. Time t is measured in seconds, and height h is measured in feet. For the ultimate effect, the fireworks must explode after they reach the maximum height. For the safety of the crowd, they must explode at least 256 ft. above the ground. The fuses must be set for the appropriate time interval that allows the fireworks to reach this height. What range of times, starting from initial launch and ending with fireworks explosion, meets these conditions?
A2.1.D Solve problems that can be represented by exponential and logarithmic functions and equations.	Examples: • If you need \$15,000 in 4 years to start college, how much money would you need to invest now? Assume an annual interest rate of 4% compounded monthly for 48 months. The half-life of a certain radioactive substance is 65 days. If there are 4.7 grams initially present, how long will it take for there to be less than 1 gram of the substance remaining?

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A2.1.E Solve problems that can be represented by inverse variations of the forms $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2}$ + b, and $f(x) = \frac{a}{(bx+c)}$.	Examples: • At the You're Toast, Dude! toaster company, the weekly cost to run the factory is $1400, and the cost of producing each toaster is an additional $4 per toaster. • Find a function to represent the weekly cost in dollars, $C(x)$, of producing x toasters. Assume either unlimited production is possible or set a maximum per week. • Find a function to represent the total production cost per toaster for a week. • How many toasters must be produced within a week to have a total production cost per toaster of $8? • A person's weight varies inversely as the square of his distance from the center of the earth. Assume the radius of the earth is 4000 miles. How much would a 200-pound man weigh 1000 miles above the surface of the earth? 2000 miles above the surface of the earth?

A2.2. Core Content: Numbers, expressions, and operations

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A2.2.C Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.	In the same way that integers were extended to fractions, polynomials are extended to rational expressions. Students must be able to perform the four basic arithmetic operations on more general expressions that involve exponentials. The binomial theorem is useful when raising expressions to powers, such as $(x + 3)^5$. Examples: • $\frac{x+1}{(x+1)^2} - \frac{3x-3}{x^2-1}$ Divide $\frac{(x+2)^{3/2}}{x+1}$ by $\frac{x+2}{x^2-1}$

A2.3. Core Content: Quadratic functions and equations

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.3.A Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.	Students translate among forms of a quadratic function to convert to one that is appropriate—e.g., vertex form—to solve specific problems. Students learn about the advantages of the standard form ($f(x) = ax^2 + bx + c$), the vertex form ($f(x) = a(x - h)^2 + d$), and the factored form ($f(x) = a(x - r)(x - s)$). They produce the vertex form by completing the square on the function in standard form, which allows them to see the symmetry of the graph of a quadratic function as well as the maximum or minimum. This opens up a whole range of new problems students can solve using quadratics. Students continue to find the solutions of the equation, which in Algebra 2 can be either real or complex. Example: • Find the minimum, the line of symmetry, and the roots for the graphs of each of the following functions: $f(x) = x^2 - 4x + 3$ $f(x) = x^2 - 4x + 4$ $f(x) = x^2 - 4x + 5$
A2.3.B Determine the number and nature of the roots of a quadratic function.	Students should be able to recognize and interpret the discriminant. Students should also be familiar with the Fundamental Theorem of Algebra, i.e., that all polynomials, not just quadratics, have roots over the complex numbers. This concept becomes increasingly important as students progress through mathematics. Example: • For what values of a does $x^2 - 6x + a$ have 2 real roots, 1 real root, and no real roots?
A2.3.C Solve quadratic equations and inequalities, including equations with complex roots.	Students solve equations that are not easily factored by completing the square and by using the quadratic formula. Examples: • $x^2 - 10x + 34 = 0$ • $3x^2 + 10 = 4x$ • Wile E. Coyote launches an anvil from 180 feet above the ground at time $t = 0$. The equation that models this situation is given by $h = -16t^2 + 96t + 180$, where t is time measured in seconds and h is height above the ground measured in feet. a. What is a reasonable domain restriction for t in this context? b. Determine the height of the anvil two seconds after it was launched. c. Determine the maximum height obtained by the anvil. d. Determine the time when the anvil is more than 100 feet above ground. • Farmer Helen wants to build a pigpen. With 100 feet of fence, she wants a rectangular pen with one side being a side of her existing barn. What dimensions should she use for her pigpen in order to have the maximum number of square feet?

A2.4. Core Content: Exponential and logarithmic functions and equations

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.4.B Graph an exponential function of the form f(x) = abx and its inverse logarithmic function.	Students expand on the work they did in Algebra 1 to functions of the form y = abx. Although the concept of inverses is not fully developed until Precalculus, there is an emphasis in Algebra 2 on students recognizing the inverse relationship between exponential and logarithmic functions and how this is reflected in the shapes of the graphs. Example: • Find the equation for the inverse function of y = 3x. Graph both functions. What characteristics of each of the graphs indicate they are inverse functions?
A2.4.C Solve exponential and logarithmic equations.	Examples: • A recommended adult dosage of the cold medication NoMoreFlu is 16 mL. NoMoreFlu causes drowsiness when there are more than 4 mL in one's system, making it unsafe to drive, operate machinery, etc. The manufacturer wants to print a warning label telling people how long they should wait after taking NoMoreFlu for the drowsiness to pass. If the typical metabolic rate is such that one quarter of the NoMoreFlu is lost every four hours, and a person takes the full dosage, how long should adults wait after taking NoMoreFlu to ensure that there will be • Less than 4 mL of NoMoreFlu in their system? • Less than 1 mL in their system? • Less than 0.1 mL in their system? • Solve for x in $256 = 2^{x^2-1}$. • Solve for x in log5(x – 4) = 3.

A2.5. Core Content: Additional functions and equations

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.5.A Construct new functions using the transformations $f(x - h)$, $f(x) + k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).	Students perform simple transformations on functions, including those that contain the absolute value of expressions, quadratic expressions, square root expressions, and exponential expressions, to make new functions. Examples: • What sequence of transformations changes $f(x) = x^2$ to $g(x) = -5(x - 3)^2 + 2$? • Carly decides to earn extra money by making glass bead bracelets. She purchases tools for \$40.00. Elastic bead cord for each bracelet costs \$0.10. Glass beads come in packs of 10 beads, and one pack has enough beads to make one bracelet. Base price for the beads is \$2.00 per pack. For each of the first 100 packs she buys, she gets \$0.01 off each of the packs. (For example, if she purchases three packs, each pack costs \$1.97 instead of \$2.00.) Carly plans to sell each bracelet for \$4.00. Assume Carly will make a maximum of 100 bracelets. Find a function $C(b)$ that describes Carly's costs. Find a function $R(b)$ that describes Carly's revenue. Carly's profit is described by $P(b) = R(b) - C(b)$. Find $P(b)$. What is the minimum number of bracelets that Carly must sell in order to make a profit? To make a profit of \$100?
A2.5.B Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x - c} + d$, and solve related equations.	Students solve algebraic equations that involve the square root of a linear expression over the real numbers. Students should be able to identify extraneous solutions and explain how they arose. Students should view the function $g(x) = \sqrt{x}$ as the inverse function of $f(x) = x^2$, recognizing that the functions have different domains for x greater than or equal to 0. Example: • Analyze the following equations and tell what you know about the solutions. Then solve the equations. $2\sqrt{x+5} = 7$ $\sqrt{5x-6} = -2$ $\sqrt{2x+15} = x$ $\sqrt{2x-5} = x+7$

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A2.5.C Plot points, sketch, and describe the graphs of functions of the form $f(x)=\frac{a}{x}+b$, $f(x)=\frac{a}{x^2}+b$, and $f(x)=\frac{a}{(bx+c)}$, and solve related equations.	Examples: • Sketch the graphs of the four functions $f(x)=\frac{a}{x^2}+b$ when a = 4 and 8 and b = 0 and 1. Sketch the graphs of the four functions $f(x)=\frac{4}{(bx+c)}$ when $b = 1$ and 4 and $c = 2$ and 3.
A2.5.D Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.	Example: • Solve for x in $60 = -2x^3 + 6$.

A2.7. Additional Key Content

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
A2.7.A Solve systems of three equations with three variables.	Students solve systems of equations using algebraic and numeric methods. Examples: • Jill, Ann, and Stan are to inherit $20,000. Stan is to get twice as much as Jill, and Ann is to get twice as much as Stan. How much does each get? • Solve the following system of equations. $2x – y – z = 7$ $3x + 5y + z = -10$ $4x – 3y + 2z = 4$
A2.7.B Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.	Students build on the knowledge gained in Algebra 1 to find specific terms in a sequence and to express arithmetic and geometric sequences in both explicit and recursive forms. Examples: • A ball is dropped from a height of 10 meters. Each time it hits the ground, it rebounds $\frac{3}{4}$ of the distance it has fallen. What is the total sum of the distances it falls and rebounds before coming to rest? • Show that the sum of the first 10 terms of the geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots$ is twice the sum of the first 10 terms of the geometric series $1 – \frac{1}{3} + \frac{1}{9} – \frac{1}{27} + \ldots$

Mathematics 1

M1.1. Core Content: Solving problems

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
M1.1.A Select and justify functions and equations to model and solve problems.	Students can analyze the rate of change of a function represented with a table or graph to determine if the function is linear. Students also analyze common ratios to determine if the function is exponential. After selecting a function to model a situation, students describe appropriate domain restrictions. They use the function to solve the problem and interpret the solution in the context of the original situation. Examples: • A cup is 6 cm tall, including a 1.1 cm lip. Find a function that represents the height of a stack of cups in terms of the number of cups in the stack. Find a function that represents the number of cups in a stack of a given height. • For the month of July, Michelle will be dog-sitting for her very wealthy, but eccentric, neighbor, Mrs. Buffett. Mrs. Buffett offers Michelle two different salary plans: Plan 1: $100 per day for the 31 days of the month. Plan 2: $1 for July 1, $2 for July 2, $4 for July 3, and so on, with the daily rate doubling each day. a. Write functions that model the amount of money Michelle will earn each day on Plan 1 and Plan 2. Justify the functions you wrote. b. State an appropriate domain for each of the models based on the context. c. Which plan should Michelle choose to maximize her earnings? Justify your recommendation mathematically. d. Extension: Write an algebraic function for the cumulative pay for each plan based on the number of days worked.
M1.1.B Solve problems that can be represented by linear functions, equations, and inequalities.	It is mathematically important to represent a word problem as an equation. Students must analyze the situation and find a way to represent it mathematically. After solving the equation, students think about the solution in terms of the original problem. Examples: • The assistant pizza maker makes 6 pizzas an hour. The master pizza maker makes 10 pizzas an hour but starts baking two hours later than his assistant. Together, they must make 92 pizzas. How many hours from when the assistant starts baking will it take? What is a general equation, in function form, that could be used to determine the number of pizzas that can be made in two or more hours? • A swimming pool holds 375,000 liters of water. Two large hoses are used to fill the pool. The first hose fills at the rate of 1,500 liters per hour and the second hose fills at the rate of 2,000 liters per hour. How many hours does it take to fill the pool completely?

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.1.C Solve problems that can be represented by a system of two linear equations or inequalities.	Examples: • An airplane flies from Baltimore to Seattle (assume a distance of 2,400 miles) in 7 hours, but the return flight takes only $4\frac{1}{2}$ hours. The air speed of the plane is the same in both directions. How many miles per hour does the plane fly with respect to the wind? What is the wind speed in miles per hour? • A coffee shop employee has one cup of 85% milk (the rest is chocolate) and another cup of 60% milk (the rest is chocolate). He wants to make one cup of 70% milk How much of the 85% milk and 60% milk should he mix together to make the 70% milk? • Two plumbing companies charge different rates for their service. Clyde's Plumbing Company charges a $75-per-visit fee that includes one hour of labor plus $45 dollars per hour after the first hour. We-Unclog-It Plumbers charges a $100-per-visit fee that includes one hour of labor plus $40 per hour after the first hour. For how many hours of plumbing work would Clyde's be less expensive than We-Unclog-It? Note: Although this context is discrete, students can model it with continuous linear functions.
M1.1.D Solve problems that can be represented by exponential functions and equations.	Students recognize common examples of exponential growth or decay, such as applying exponential functions to determine compound interest, population growth, and radioactivity. They approximate solutions with graphs or tables, check solutions numerically, and when possible, solve problems exactly. Example: • Mr. Tsu invests $1000 in a 5-year CD that pays 4% interest compounded yearly. Present to Mr. Tsu his expected balance at the end of years 1, 3, and 5 and the process you used to arrive at each value.

M1.2. Core Content: Characteristics and behaviors of functions

Performance Expectation *Students are expected to:*	*Explanatory Comments and Examples*
M1.2.A Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.	Functions studied in Mathematics 1 include linear and those defined piecewise (including step functions and those that contain the absolute value of an expression). They compare and contrast non-linear functions, such as quadratic and exponential, to linear functions.

Given a problem situation, students should describe further restrictions on the domain of a function that are appropriate for the problem context.

Examples:

- Which of the following are functions? Explain why or why not.
  - — The age in years of each student in your math class and each student's shoe size.
  - — The number of degrees a person rotates a spigot and the volume of water that comes out of the spigot.
- A function $f(n) = 60n$ is used to model the distance in miles traveled by a car traveling 60 miles per hour in n hours. Identify the domain and range of this function. What restrictions on the domain of this function should be considered for the model to correctly reflect the situation?
- What is the domain of $f(x) = \sqrt{5 - x}$?
- Which of the following equations, inequalities, or graphs determine y as a function of x?

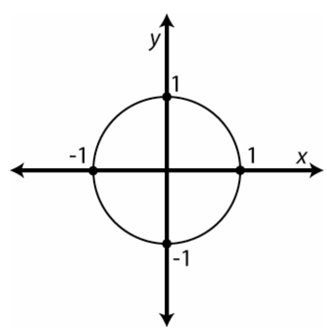
  $y = 2$

  $x = 3$

  $y = |x|$

  $y = \begin{cases} x + 3, & x \le 1 \\ x - 2, & x > 1 \end{cases}$

  $x^2 + y^2 = 1$

- 

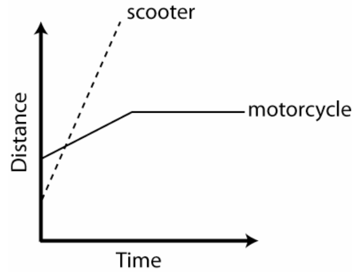
Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.2.B Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.	This expectation applies each time a new class (family) of functions is encountered. In Mathematics 1, students should be introduced to a variety of additional functions that include expressions such as x^3, $\sqrt{x}$, $\frac{1}{x}$, and absolute values. They will study these functions in depth in subsequent courses. Students should know that f(x) = $\frac{a}{x}$ represents an inverse variation. Students begin to describe the graph of a function from its symbolic expression, and use key characteristics of the graph of a function to infer properties of the related symbolic expression. Translating among these various representations of functions is an important way to demonstrate conceptual understanding of functions. Students learn that each representation has particular advantages and limitations. For example, a graph shows the shape of a function, but not exact values. They also learn that a table of values may not uniquely determine a single function without some specification of the nature of that function (e.g., it is quadratic).
M1.2.C Evaluate *f*(*x*) at *a* (i.e., *f*(*a*)) and solve for *x* in the equation *f*(*x*) = *b*.	Functions may be described and evaluated with symbolic expressions, tables, graphs, or verbal descriptions. Students should distinguish between solving for f(x) and evaluating a function at x. Example: • Roses-R-Red sells its roses for $0.75 per stem and charges a $20 delivery fee per order. — What is the cost of having 10 roses delivered? How many roses can you have delivered for $65?
M1.2.D Plot points, sketch, and describe the graphs of functions of the form f(x) = $\frac{a}{x}$ + b.	Mathematics 1 addresses only rational functions of the form f(x) = $\frac{a}{x}$ + b. Rational functions of the form f(x) = $\frac{a}{x^2}$ + b and f(x) = $\frac{a}{(bx+c)}$ are addressed in Mathematics 3. Example: • Sketch the graphs of the four functions *f*(*x*) = $\frac{a}{x}$ + *b* when *a* = 4 and 8 and *b* = 0 and 1.

M1.3. Core Content: Linear functions, equations, and relationships

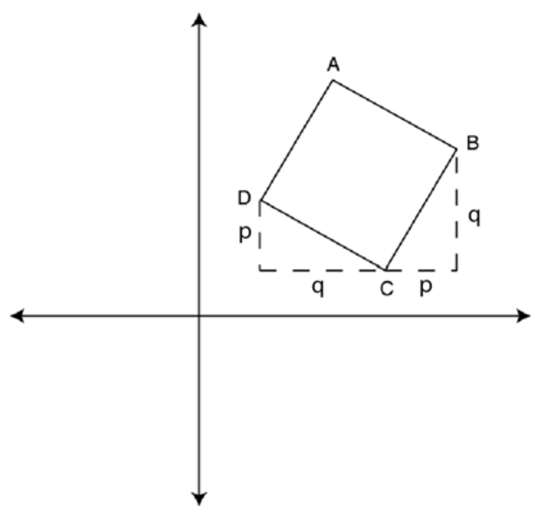
Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
M1.3.A Write and solve linear equations and inequalities in one variable.	This expectation includes the use of absolute values in the equations and inequalities. Examples: • Write an absolute value equation or inequality for all the numbers 2 units from 7, and all the numbers that are more than *b* units from *a*. • Solve $\lvert x - 6\rvert \le 4$ and locate the solution on the number line. • Write an equation or inequality that has no real solutions; infinite numbers of real solutions; and exactly one real solution. • Solve for *x* in $2(x - 3) + 4x = 15 + 2x$. • Solve $8.5 < 3x + 2 \le 9.7$ and locate the solution on the number line.
M1.3.B Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.	In the case of a linear function $y = f(x)$, expressed in slope-intercept form ($y = mx + b$), *m* and *b* are parameters. Students should know that $f(x) = kx$ represents a direct variation (proportional relationship). Examples: • Graph a function of the form $f(x) = kx$, describe the effect that changes on *k* have on the graph and on $f(x)$, and answer questions that arise in proportional situations. • A gas station's 10,000-gallon underground storage tank contains 1,000 gallons of gasoline. Tanker trucks pump gasoline into the tank at a rate of 400 gallons per minute. How long will it take to fill the tank? Find a function that represents this situation and then graph the function. If the flow rate increases from 400 to 500 gallons per minute, how will the graph of the function change? If the initial amount of gasoline in the tank changes from 1,000 to 2,000 gallons, how will the graph of the function change? • Compare and contrast the functions $y = 3\lvert x\rvert$ and $y = -\frac{1}{3}\lvert x\rvert$.

Performance Expectations *Students are expected to:*	Explanatory Comments and Examples
M1.3.C Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.	Examples:

- The graph shows the relationship between time and distance from a gas station for a motorcycle and a scooter. What can be said about the relative speed of the motorcycle and scooter that matches the information in the graph? What can be said about the intersection of the graphs of the scooter and the motorcycle? Is it possible to tell which vehicle is further from the gas station at the initial starting point represented in the graph? At the end of the time represented in the graph? Why or why not?



- A 1,500-gallon tank contains 200 gallons of water. Water begins to run into the tank at the rate of 75 gallons per hour. When will the tank be full? Find a linear function that models this situation, draw a graph, and create a table of data points. Once you have answered the question and completed the tasks, explain your reasoning. Interpret the slope and *y*-intercept of the function in the context of the situation.
- Given that the figure below is a square, find the slope of the perpendicular sides AB and BC. Describe the relationship between the two slopes.



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Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.3.D Write and graph an equation for a line given the slope and the *y*-intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.	Linear equations may be written in slope-intercept, point-slope, and standard form. Examples: • Find an equation for a line with *y*-intercept equal to 2 and slope equal to 3. • Find an equation for a line with a slope of 2 that goes through the point (1, 1). • Find an equation for a line that goes through the points (-3, 5) and (6, -2). • For each of the following, use only the equation (without sketching the graph) to describe the graph. — $y = 2x + 3$ — $y - 7 = 2(x - 2)$ • Write the equation $3x + 2y = 5$ in slope intercept form. • Write the equation $y - 1 = 2(x - 2)$ in standard form.
M1.3.F Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and y-intercept of the line, and use the equation to make predictions.	Students need to be able to evaluate the quality of their predictions, recognizing that extrapolation is based on the assumption that the trend indicated continues beyond the unknown data.

M1.6. Core Content: Numbers, expressions, and operations

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.6.C Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.	Students learn to use letters as variables and in other ways that increase in sophistication throughout high school. For example, students learn that letters can be used: • To represent fixed and temporarily unknown values in equations, such as $3x + 2 = 5$; • To express identities, such as $x + x = 2x$ for all *x*; • As attributes in formulas, such as $A = lw$; • As constants such as *a*, *b*, and *c* in the equation $y = ax^2 + bx + c$; • As parameters in equations, such as the *m* and *b* for the family of functions defined by $y = mx + b$; • To represent varying quantities, such as *x* in $f(x) = 5x$; • To represent functions, such as *f* in $f(x) = 5x$; and • To represent specific numbers, such as π.

Expressions include those involving polynomials, radicals, absolute values, and integer exponents.

Examples:

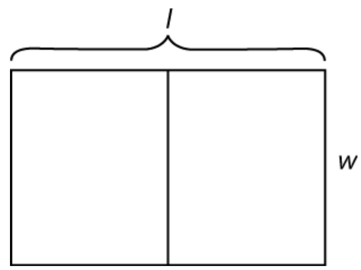
- For what values of *a* and *n*, where *n* is an integer greater than 0, is a^n always negative?
- For what values of *a* is $\frac{1}{a}$ an integer?
- For what values of *a* is $\sqrt{5 - a}$ defined?
- For what values of *a* is -*a* always positive?

M1.7. Additional Key Content:

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.7.A Sketch the graph for an exponential function of the form $y = ab^n$ where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.	Examples: • Sketch the graph of $y = 2^n$ by hand. • You have won a door prize and are given a choice between two options: • $150 invested for 10 years at 4% compounded annually. • $200 invested for 10 years at 3% compounded annually. • How much is each worth at the end of each year of the investment periods? • Are the two investments ever equal in value? Which will you choose?
M1.7.B Find and approximate solutions to exponential equations.	Students can approximate solutions using graphs or tables with and without technology.

Mathematics 2

M2.1. Core Content: Modeling situations and solving problems

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.1.A Select and justify functions and equations to model and solve problems.	Example: • Dawson wants to make a horse corral by creating a rectangle that is divided into 2 parts, similar to the following diagram. He has a 1200-foot roll of fencing to do the job. What are the dimensions of the enclosure with the largest total area? What function or equation best models this situation? 
M2.1.B Solve problems that can be represented by systems of equations and inequalities.	Example: • Data derived from an experiment seems to be parabolic when plotted on a coordinate grid. Three observed data points are (2, 10), (3, 8), and (4, 4). Write a quadratic equation that passes through the points.
M2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.	Students solve problems by factoring and applying the quadratic formula to the quadratic equation, and use the vertex form of the equation to solve problems about maximums, minimums, and symmetry. Examples: • Find the solutions to the simultaneous equations $y = x + 2$ and $y = x^2$. • If you throw a ball straight up (with initial height of 4 feet) at 10 feet per second, how long will it take to fall back to the starting point? The function $h(t) = -16t^2 + v_0t + h_0$ describes the height, h in feet, of an object after t seconds, with initial velocity v_0 and initial height h_0. • Joe owns a small plot of land 20 feet by 30 feet. He wants to double the area by increasing both the length and the width, keeping the dimensions in the same proportion as the original. What will be the new length and width? • What two consecutive numbers, when multiplied together, give the first number plus 16? Write the equation that represents the situation. • The Gateway Arch in St. Louis has a special shape called a catenary, which looks a lot like a parabola. It has a base width of 600 feet and is 630 feet high. Which is taller, this catenary arch or a parabolic arch that has the same base width but has a height of 450 feet at a point 150 feet from one of the pillars? What is the height of the parabolic arch?

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***

M2.1.D Solve problems that can be represented by exponential functions and equations.

Students extend their use of exponential functions and equations to solve more complex problems. They approximate solutions with graphs or tables, check solutions numerically, and when possible, solve problems exactly.

Examples:

- E. coli bacteria reproduce by a simple process called binary fission—each cell increases in size and divides into two cells. In the laboratory, E. coli bacteria divide approximately every 15 minutes. A new E. coli culture is started with 1 cell.
  - d. Find a function that models the E. coli population size at the end of each 15-minute interval. Justify the function you found.
  - e. State an appropriate domain for the model based on the context.
  - f. After what 15-minute interval will you have at least 500 bacteria?
- Estimate the solution to $2^x = 16{,}384$

M2.2. Quadratic functions, equations, and relationships:

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***

M2.2.A Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.

Example:

- Kendre and Tyra built a tennis ball cannon that launches tennis balls straight up in the air at an initial velocity of 50 feet per second. The mouth of the cannon is 2 feet off the ground. The function $h(t) = -16t^2 + 50t + 2$ describes the height, h, in feet, of the ball t seconds after the launch.

  Make a table from the function. Then use the table to sketch a graph of the height of the tennis ball as a function of time into the launch. Give a verbal description of the graph. How high was the ball after 1 second? When does it reach this height again?

Performance Expectations *Students are expected to:*	*Explanatory Comments and Examples*
M2.2.B Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the *x*-intercepts as solutions to a quadratic equation.	Note that in Mathematics 2, the parameter b in the term bx in the quadratic form $ax^2 + bx + c$ is not often used to provide useful information about the characteristics of the graph. Parameters considered most useful are: • a and c in $f(x) = ax^2 + c$ • a, h, and k in $f(x) = a(x - h)^2 + k$, and • a, r, and s in $f(x) = a(x - r)(x - s)$ Example: • A particular quadratic function can be expressed in the following two ways: $f(x) = -(x - 3)^2 + 1$ $f(x) = -(x - 2)(x - 4)$ What information about the graph can be directly inferred from each of these forms? Explain your reasoning. Sketch the graph of this function, showing the roots.
M2.2.C Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.	Students translate among forms of a quadratic function to convert to one that is appropriate—e.g., vertex form—to solve specific problems. Students learn about the advantages of the standard form ($f(x) = ax^2 + bx + c$), the vertex form ($f(x) = a(x - h)^2 + d$), and the factored form ($f(x) = a(x - r)(x - s)$). They produce the vertex form by completing the square on the function in standard form, which allows them to see the symmetry of the graph of a quadratic function as well as the maximum or minimum. This opens up a whole range of new problems students can solve using quadratics. Students continue to find the solutions of the equation, which can be either real or complex. Example: • Find the minimum, the line of symmetry, and the roots for the graphs of each of the following functions: $f(x) = x^2 - 4x + 3$ $f(x) = x^2 - 4x + 4$ $f(x) = x^2 - 4x + 5$

Performance Expectations	***Explanatory Comments and Examples***
Students are expected to:	

M2.2.D Solve quadratic equations that can be factored as (ax + b)(cx + d) where a, b, c, and d are integers.

Students learn to efficiently solve quadratic equations by recognizing and using the simplest factoring methods, including recognizing special quadratics as squares and differences of squares.

Examples:

- $2x^2 + x - 3 = 0$; $(x - 1)(2x + 3) = 0$; $x = 1, -\frac{3}{2}$
- $4x^2 + 6x = 0$; $2x(2x + 3) = 0$; $x = 0$, Š$\frac{3}{2}$
- $36x^2 - 25 = 0$; $(6x + 5)(6x - 5) = 0$; $x = \pm\frac{5}{6}$
- $x^2 + 6x + 9 = 0$; $(x + 3)^2 = 0$; $x = -3$

M2.2.E Determine the number and nature of the roots of a quadratic function.

Students should be able to recognize and interpret the discriminant.

Students should also be familiar with the Fundamental Theorem of Algebra, i.e., that all polynomials, not just quadratics, have roots over the complex numbers. This concept becomes increasingly important as students progress through mathematics.

Example:

- For what values of *a* does $x^2 - 6x + a$ have 2 real roots, 1 real root, and no real roots?

M2.2.F Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.

Students solve those equations that are not easily factored by completing the square and by using the quadratic formula. Completing the square should also be used to derive the quadratic formula.

Students learn how to determine if there are two, one, or no real solutions.

Examples:

- Complete the square to solve $x^2 + 4x = 13$.

  $x^2 + 4x - 13 = 0$

  $x^2 + 4x + 4 = 17$

  $(x + 2)^2 = 17$

  $x + 2 = \pm\sqrt{17}$

  $x = -2 \pm\sqrt{17}$

  $x \approx 2.12, -6.12$

  Use the quadratic formula to solve $4x^2 - 2x = 5$.

  $x \approx 1.40, -0.90$

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.2.G Solve quadratic equations and inequalities, including equations with complex roots.	Students solve equations that are not easily factored by completing the square and by using the quadratic formula. Examples: • $x^2 - 10x + 34 = 0$ • $3x^2 + 10 = 4x$ • Wile E. Coyote launches an anvil from 180 feet above the ground at time t = 0. The equation that models this situation is given by $h = -16t^2 + 96t + 180$, where t is time measured in seconds and h is height above the ground measured in feet. a. What is a reasonable domain restriction for t in this context? b. Determine the height of the anvil two seconds after it was launched. c. Determine the maximum height obtained by the anvil. d. Determine the time when the anvil is more than 100 feet above ground. • Farmer Helen wants to build a pigpen. With 100 feet of fence, she wants a rectangular pen with one side being a side of her existing barn. What dimensions should she use for her pigpen in order to have the maximum number of square feet?
M2.2.H Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	In high school, determining a formula for a curve of best fit requires a graphing calculator or similar technological tool.

M2.5. Additional Key Content:

Performance Expectations Students are expected to:	Explanatory Comments and Examples
M2.5.A Use algebraic properties to factor and combine like terms in polynomials.	Algebraic properties include the commutative, associative, and distributive properties. Factoring includes: • Factoring a monomial from a polynomial, such as $4x^2 + 6x = 2x(2x + 3)$; • Factoring the difference of two squares, such as $36x^2 – 25y^2 = (6x + 5y)(6x – 5y)$ and $x^4 – y^4 = (x + y)(x – y)(x^2 + y^2)$; • Factoring perfect square trinomials, such as $x^2 + 6xy + 9y^2 = (x + 3y)^2$; • Factoring quadratic trinomials such as $x^2 + 5x + 4 = (x + 4)(x + 1)$; and • Factoring trinomials that can be expressed as the product of a constant and a trinomial, such as $0.5x^2 – 2.5x – 7 = 0.5(x + 2)(x – 7)$.
M2.5.D Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.	Students build on the knowledge gained in Mathematics 1 to find specific terms in a sequence and to express arithmetic and geometric sequences in both explicit and recursive forms. Examples: • A ball is dropped from a height of 10 meters. Each time it hits the ground, it rebounds $\frac{3}{4}$ of the distance it has fallen. What is the total sum of the distances it falls and rebounds before coming to rest? • Show that the sum of the first 10 terms of the geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ...$ is twice the sum of the first 10 terms of the geometric series $1 – \frac{1}{3} + \frac{1}{9} – \frac{1}{27} + ...$

Mathematics 3

M3.1. Core Content: Solving problems

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.1.A Select and justify functions and equations to model and solve problems.	Examples: • A manufacturer wants to design a cylindrical soda can that will hold 500 milliliters (mL) of soda. The manufacturer's research has determined that an optimal can height is between 10 and 15 centimeters. Find a function for the radius in terms of the height, and use it to find the possible range of radius measurements in centimeters. Explain your reasoning.
M3.1.B Solve problems that can be represented by systems of equations and inequalities.	Examples: • Mr. Smith uses the following formula to calculate students' final grades in his Mathematics 3 class: $0.4E + 0.6T = C$, where E represents the score on the final exam, and T represents the average score of all tests given during the grading period. All tests and the final exam are worth a maximum of 100 points. The minimum passing score on tests, the final exam, and the course is 60. Determine the inequalities that describe the following situation and sketch a system of graphs to illustrate it. When necessary, round scores to the nearest tenth. — Is it possible for a student to have a failing test score average (i.e., $T < 60$ points) and still pass the course? — If you answered "yes," what is the minimum test score average a student can have and still pass the course? What final exam score is needed to pass the course with a minimum test score average? — A student has a particular test score average. How can (s)he figure out the minimum final exam score needed to pass the course?
M3.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.	In addition to solving area and velocity problems by factoring and applying the quadratic formula to the quadratic equation, students use the vertex form of the equation to solve problems about maximums, minimums, and symmetry. Examples: • Fireworks are launched upward from the ground with an initial velocity of 160 feet per second. The formula for vertical motion is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second, v represents the initial velocity, and s represents the initial height. Time t is measured in seconds, and height h is measured in feet. For the ultimate effect, the fireworks must explode after they reach the maximum height. For the safety of the crowd, they must explode at least 256 ft. above the ground. The fuses must be set for the appropriate time interval that allows the fireworks to reach this height. What range of times, starting from initial launch and ending with fireworks explosion, meets these conditions?

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

M3.1.D Solve problems that can be represented by exponential and logarithmic functions and equations.

Examples:

- If you need \$15,000 in 4 years to start college, how much money would you need to invest now? Assume an annual interest rate of 4% compounded monthly for 48 months.
- The half-life of a certain radioactive substance is 65 days. If there are 4.7 grams initially present, how long will it take for there to be less than 1 gram of the substance remaining?

M3.1.E Solve problems that can be represented by inverse variations of the forms $f(x)=\frac{a}{x}+b$, $f(x)=\frac{a}{x^2}+b$, and $f(x)=\frac{a}{(bx+c)}$.

Examples:

- At the You're Toast, Dude! toaster company, the weekly cost to run the factory is \$1400, and the cost of producing each toaster is an additional \$4 per toaster.
  - — Find a function to represent the weekly cost in dollars, $C(x)$, of producing x toasters. Assume either unlimited production is possible or set a maximum per week.
  - — Find a function to represent the total production cost per toaster for a week.
  - — How many toasters must be produced within a week to have a total production cost per toaster of \$8?
- A person's weight varies inversely as the square of his distance from the center of the earth. Assume the radius of the earth is 4000 miles. How much would a 200-pound man weigh
  - — 1000 miles above the surface of the earth?
  - — 2000 miles above the surface of the earth?
  - —

M3.2. Core Content: Transformations and functions

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.2.E Construct new functions using the transformations f(x – h), f(x) + k, cf(x), and by adding and subtracting functions, and describe the effect on the original graph(s).	Students perform simple transformations on functions, including those that contain the absolute value of expressions, quadratic expressions, square root expressions, and exponential expressions, to make new functions. Examples: • What sequence of transformations changes $f(x) = x^2$ to $g(x) = -5(x - 3)^2 + 2$? • Carly decides to earn extra money by making glass bead bracelets. She purchases tools for $40.00. Elastic bead cord for each bracelet costs $0.10. Glass beads come in packs of 10 beads, and one pack has enough beads to make one bracelet. Base price for the beads is $2.00 per pack. For each of the first 100 packs she buys, she gets $0.01 off each of the packs. (For example, if she purchases three packs, each pack costs $1.97 instead of $2.00.) Carly plans to sell each bracelet for $4.00. Assume Carly will make a maximum of 100 bracelets. Find a function $C(b)$ that describes Carly's costs. Find a function $R(b)$ that describes Carly's revenue. Carly's profit is described by $P(b) = R(b) - C(b)$. Find $P(b)$. What is the minimum number of bracelets that Carly must sell in order to make a profit? To make a profit of $100?

M3.3. Core Content: Functions and modeling

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.3.B Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.	Students expand on the work they did in Mathematics 2 with functions of the form $y = ab^x$. Although the concept of inverses is not fully developed until Precalculus, there is an emphasis in Mathematics 3 on students recognizing the inverse relationship between exponential and logarithmic functions and how this is reflected in the shapes of the graphs. Example: Find the equation for the inverse function of $y = 3^x$. Graph both functions. What characteristics of each of the graphs indicate they are inverse functions?

Performance Expectations	***Explanatory Comments and Examples***
Students are expected to:	
M3.3.C Solve exponential and logarithmic equations.	Examples: • A recommended adult dosage of the cold medication NoMoreFlu is 16 mL. NoMoreFlu causes drowsiness when there are more than 4 mL in one's system, making it unsafe to drive, operate machinery, etc. The manufacturer wants to print a warning label telling people how long they should wait after taking NoMoreFlu for the drowsiness to pass. If the typical metabolic rate is such that one quarter of the NoMoreFlu is lost every four hours, and a person takes the full dosage, how long should adults wait after taking NoMoreFlu to ensure that there will be — Less than 4 mL of NoMoreFlu in their system? — Less than 1 mL in their system? — Less than 0.1 mL in their system? • Solve for x in $256 = 2^{x^2-1}$. • Solve for x in $\log_5(x - 4) = 3$.
M3.3.D Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x-c} + d$, and solve related equations.	Students solve algebraic equations that involve the square root of a linear expression over the real numbers. Students should be able to identify extraneous solutions and explain how they arose. Students should view the function $g(x) = \sqrt{x}$ as the inverse function of $f(x) = x^2$, recognizing that the functions have different domains for x greater than or equal to 0. Example: • Analyze the following equations and tell what you know about the solutions. Then solve the equations. — $2\sqrt{x+5} = 7$ — $\sqrt{5x-6} = -2$ — $\sqrt{2x+15} = x$ — $\sqrt{2x-5} = x+7$
M3.3.E Plot points, sketch, and describe the graphs of functions of the form $f(x) = \frac{a}{x^2} + b$ and $f(x) = \frac{a}{(bx+c)}$, and solve related equations.	Examples: • Sketch the graphs of the four functions $f(x) = \frac{a}{x^2} + b$ when a = 4 and 8 and b = 0 and 1. • Sketch the graphs of the four functions $f(x) = \frac{4}{(bx+c)}$ when b = 1 and 4 and c = 2 and 3.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.3.F Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.	Example: • Solve for x in 60 = -2x3 + 6.
M3.3.G Solve systems of three equations with three variables.	Students solve systems of equations using algebraic and numeric methods. Examples: • Jill, Ann, and Stan are to inherit $20,000. Stan is to get twice as much as Jill, and Ann is to get twice as much as Stan. How much does each get? • Solve the following system of equations. $2x - y - z = 7$ $3x + 5y + z = -10$ $4x - 3y + 2z = 4$

M3.6. Core Content: Algebraic properties

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.6.D Add, subtract, multiply, divide, and simplify rational and **more** general algebraic expressions.	In the same way that integers were extended to fractions, polynomials are extended to rational expressions. Students must be able to perform the four basic arithmetic operations on more general expressions that involve exponentials. The binomial theorem is useful when raising expressions to powers, such as (x + 3)5. Examples: • $\frac{x+1}{(x+1)^2} - \frac{3x-3}{x^2-1}$ • Divide $\frac{(x+2)^{3/2}}{x+1}$ by $\frac{x+2}{x^2-1}$

Geometry/Measurement Strand

In kindergarten through grade 2, students become familiar with locations in space and describe and compare two- and three-dimensional figures. They compare objects and begin to learn what it means to measure something, beginning with length. Students in grades 3–5 continue to learn about special characteristics of geometric figures, including symmetry and congruence. They compute the perimeters and areas of figures, and they develop volume concepts by computing volumes of rectangular prisms. In grades 6–8, students connect what they know about measurement skills to geometric ideas, solving problems involving π, the Pythagorean Theorem, and similarity. **By the end of high school, students know and can prove properties of two- and three-dimensional geometric figures, and they solve increasingly complex problems involving geometric figures and their measurements**.

This is one of six strand documents that accompany the Washington State K–8 Mathematics Standards, tracking the development of important mathematical ideas and skills across grades K–8. Where content of an expectation may address more than one strand, that expectation may appear in more than one strand document.

Geometry

G.1. Core Content: Logical arguments and proofs

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
G.1.A Distinguish between inductive and deductive reasoning.	Students generate and test conjectures inductively and then prove (or disprove) their conclusions deductively. Example: • A student first hypothesizes that the number of degrees in a polygon = 180 · (*s* – 2), where *s* represents the number of sides, and then proves this is true. When was the student using inductive reasoning? When was s/he using deductive reasoning? Justify your answers.
G.1.B Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.	Examples: • Investigate the relationship among the medians of a triangle using paper folding. Make a conjecture about this relationship. • Using dynamic geometry software, decide if the following is a plausible conjecture: If segment AM is a median in triangle ABC, then ray AM bisects angle BAC.
G.1.C Use deductive reasoning to prove that a valid geometric statement is true.	Valid proofs may be presented in paragraph, two-column, or flow-chart formats. Proof by contradiction is a form of deductive reasoning. Example: • Prove that the diagonals of a rhombus are perpendicular bisectors of each other.
G.1.D Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.	Examples: • If *m* and *n* are odd integers, then the sum of *m* and *n* is an even integer. State the converse and determine whether it is valid. • If a quadrilateral is a rectangle, the diagonals have the same length. State the contrapositive and determine whether it is valid.
G.1.E Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.	Example: • Identify errors in reasoning in the following proof: Given $\angle ABC \cong \angle PRQ$, $\overline{AB} \cong \overline{PQ}$, and $\overline{BC} \cong \overline{QR}$, then $\Delta ABC \cong \Delta PQR$ by SAS.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
G.1.F Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.	Students sketch *points* and *lines* (undefined terms) and define and sketch representations of other common terms. They use definitions and postulates as they prove theorems throughout geometry. In their work with theorems, they identify the hypothesis and the conclusion and explain the role of each. Students describe the consequences of changing assumptions or using different definitions for subsequent theorems and logical arguments. Example: • There are two definitions of trapezoid that can be found in books or on the web. A trapezoid is either a quadrilateral with exactly one pair of parallel sides or a quadrilateral with at least one pair of parallel sides. • Write some theorems that are true when applied to one definition but not the other, and explain your answer.

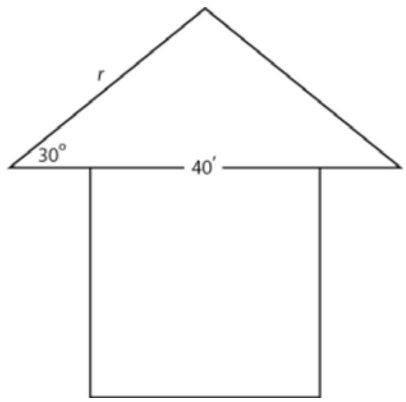
G.2. Core Content: Lines and angles

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
G.2.A Know, prove, and apply theorems about parallel and perpendicular lines.	Students should be able to summarize and explain basic theorems. They are not expected to recite lists of theorems, but they should know the conclusion of a theorem when given its hypothesis. Examples: • Prove that a point on the perpendicular bisector of a line segment is equidistant from the ends of the line segment. • If each of two lines is perpendicular to a given line, what is the relationship between the two lines? How do you know?
G.2.B Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.	Examples: • Prove that if two parallel lines are cut by a transversal, then alternate-interior angles are equal. • Take two parallel lines l and m, with (distinct) points A and B on l and C and D on m. If $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{BD}$ at point E, prove that $\Delta ABE \approx \Delta CDE$.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.2.C Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.	Constructions using circles and lines with dynamic geometry software (i.e., virtual compass and straightedge) are equivalent to paper and pencil constructions. Example: • Construct and mathematically justify the steps to: • Bisect a line segment. • Drop a perpendicular from a point to a line. • Construct a line through a point that is parallel to another line.
G.2.D Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.	Example: • Describe all the ways that three planes can intersect in space.

G.3. Core Content: Two- and three dimensional figures

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.3.A Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.	Examples: • Prove that the sum of the angles of a triangle is 180°. • Prove and explain theorems about the incenter, circumcenter, orthocenter, and centroid. • The rural towns of Atwood, Bridgeville, and Carnegie are building a communications tower to serve the needs of all three towns. They want to position the tower so that the distance from each town to the tower is equal. Where should they locate the tower? How far will it be from each town?
G.3.B Determine and prove triangle congruence, triangle similarity, and other properties of triangles.	Students should identify necessary and sufficient conditions for congruence and similarity in triangles, and use these conditions in proofs. Examples: • Prove that congruent triangles are similar. • For a given ΔRST, prove that ΔXYZ, formed by joining the midpoints of the sides of ΔRST, is similar to ΔRST. • Show that not all SSA triangles are congruent.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
G.3.C Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.	Examples: • Determine the length of the altitude of an equilateral triangle whose side lengths measure 5 units. • If one leg of a right triangle has length 5 and the adjacent angle is 30°, what is the length of the other leg and the hypotenuse? • If one leg of a 45°–45°–90° triangle has length 5, what is the length of the hypotenuse? • The pitch of a symmetrical roof on a house 40 feet wide is 30º. What is the length of the rafter, *r*, exactly and approximately?  •

Performance Expectation

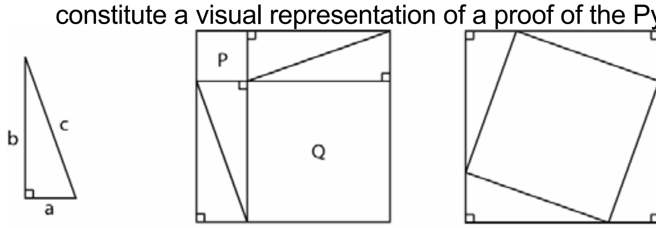
Students are expected to:

Explanatory Comments and Examples

G.3.D Know, prove, and apply the Pythagorean Theorem and its converse.

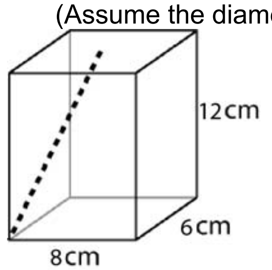
Examples:

- Consider any right triangle with legs *a* and *b* and hypotenuse *c*. The right triangle is used to create Figures 1 and 2. Explain how these figures constitute a visual representation of a proof of the Pythagorean Theorem.



Figures 1 Figure 2

- A juice box is 6 cm by 8 cm by 12 cm. A straw is inserted into a hole in the center of the top of the box. The straw must stick out 2 cm so you can drink from it. If the straw must be long enough to touch each bottom corner of the box, what is the minimum length the straw must be? (Assume the diameter of the straw is 0 for the mathematical model.)



- In △ABC, with right angle at *C*, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

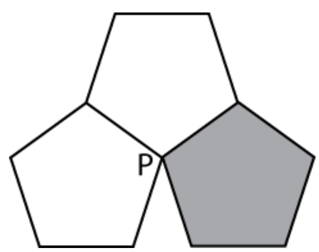
- Apply the Pythagorean Theorem to derive the distance formula in the (*x*, *y*) plane.

G.3.E Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.

Examples:

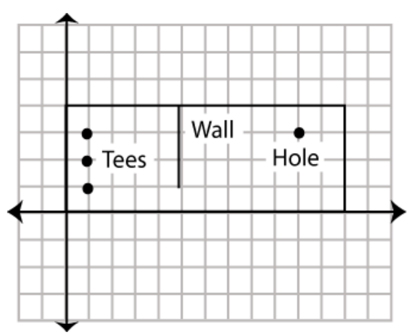
- A 12-foot ladder leans against a wall to form a 63° angle with the ground. How many feet above the ground is the point on the wall at which the ladder is resting?

- Use the Pythagorean Theorem to establish that $\sin^2ø + \cos^2ø = 1$ for ø between 0° and 90°.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.3.F Know, prove, and apply basic theorems about parallelograms.	Properties may include those that address symmetry and properties of angles, diagonals, and angle sums. Students may use inductive and deductive reasoning and counterexamples. Examples: • Are opposite sides of a parallelogram always congruent? Why or why not? • Are opposite angles of a parallelogram always congruent? Why or why not? • Prove that the diagonals of a parallelogram bisect each other. • Explain why if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. • Prove that the diagonals of a rectangle are congruent. Is this true for any parallelogram? Justify your reasoning.
G.3.G Know, prove, and apply theorems about properties of quadrilaterals and other polygons.	Examples: • What is the length of the apothem of a regular hexagon with side length 8? What is the area of the hexagon? • If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the number of sides, *n*, of a replacement polygon that makes the three polygons fit perfectly. 
G.3.H Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.	Examples: • Given a line tangent to a circle, know and explain that the line is perpendicular to the radius drawn to the point of tangency. • Prove that two chords equally distant from the center of a circle are congruent. • Prove that if one side of a triangle inscribed in a circle is a diameter, then the triangle is a right triangle. • Prove that if a radius of a circle is perpendicular to a chord of a circle, then the radius bisects the chord.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.3.I Explain and perform constructions related to the circle.	Students perform constructions using straightedge and compass, paper folding, and dynamic geometry software. What is important is that students understand the mathematics and are able to justify each step in a construction. Example: • In each case, explain why the constructions work: a. Construct the center of a circle from two chords. b. Construct a circumscribed circle for a triangle. c. Inscribe a circle in a triangle.
G.3.J Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.	Examples: • Given the number of faces of a regular polyhedron, derive a formula for the number of vertices. • Describe symmetries of three-dimensional polyhedra and their two-dimensional faces. • Describe the lateral faces that are required for a pyramid to be a right pyramid with a regular base. Describe the lateral faces required for an oblique pyramid that has a regular base.
G.3.K Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.	Examples: • Start with a regular tetrahedron with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the tetrahedron is a square. Find the area of the square. (Requires no pencil or paper.) • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a regular hexagon. Find the area of the hexagon. • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a rectangle that is not a face and contains four of the vertices. Find the area of the rectangle. • Which has the larger area, the above rectangle or the above hexagon?

Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.4.A	Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.	Examples: • Write an equation for the perpendicular bisector of a given line segment. • Determine the equation of a line through the points (5, 3) and (5, -2). • Prove that the slopes of perpendicular lines are negative inverses of each other.
G.4.B	Determine the coordinates of a point that is described geometrically.	Examples: • Determine the coordinates for the midpoint of a given line segment. • Given the coordinates of three vertices of a parallelogram, determine all possible coordinates for the fourth vertex. • Given the coordinates for the vertices of a triangle, find the coordinates for the center of the circumscribed circle and the length of its radius.
G.4.C	Verify and apply properties of triangles and quadrilaterals in the coordinate plane.	Examples: • Given four points in a coordinate plane that are the vertices of a quadrilateral, determine whether the quadrilateral is a rhombus, a square, a rectangle, a parallelogram, or none of these. Name all the classifications that apply. • Given a parallelogram on a coordinate plane, verify that the diagonals bisect each other. • Given the line with *y*-intercept 4 and *x*-intercept 3, find the area of a square that has one corner on the origin and the opposite corner on the line described. • Below is a diagram of a miniature golf hole as drawn on a coordinate grid. The dimensions of the golf hole are 4 feet by 12 feet. Players must start their ball from one of the three tee positions, located at (1, 1), (1, 2), or (1, 3). The hole is located at (10, 3). A wall separates the tees from the hole. At which tee should the ball be placed to create the shortest "hole-in-one" path? Sketch the intended path of the ball, find the distance the ball will travel, and describe your reasoning. (Assume the diameters of the golf ball and the hole are 0 for the mathematical model.)



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Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.4.D Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).	Examples: • Write an equation for a circle with a radius of 2 units and center at (1, 3). • Given the circle $x^2 + y^2 = 4$ and the line $y = x$, find the points of intersection. • Write an equation for a circle given a line segment as a diameter. • Write an equation for a circle determined by a given center and tangent line.

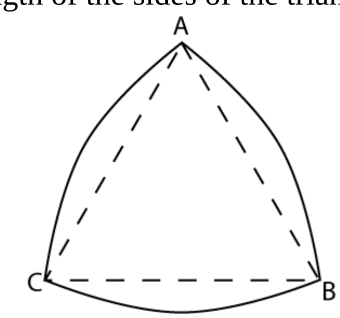
G.5. Core Content: Geometric transformations

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.5.A Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.	Transformations include translations, rotations, reflections, and dilations. Example: • Line *m* is described by the equation $y = 2x + 3$. Graph line *m* and reflect line *m* across the line $y = x$. Determine the equation of the image of the reflection. Describe the relationship between the line and its image.
G.5.B Determine and apply properties of transformations.	Students make and test conjectures about compositions of transformations and inverses of transformations, the commutativity and associativity of transformations, and the congruence and similarity of two-dimensional figures under various transformations. Examples: • Identify transformations (alone or in composition) that preserve congruence. • Determine whether the composition of two reflections of a line is commutative. • Determine whether the composition of two rotations about the same point of rotation is commutative. • Find a rotation that is equivalent to the composition of two reflections over intersecting lines. • Find the inverse of a given transformation.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.5.C Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.	Examples: • Find a sequence of transformations that superimposes the segment with endpoints (0, 0) and (2, 1) on the segment with endpoints (4, 2) and (3, 0). • Find a sequence of transformations that superimposes the triangle with vertices (0, 0), (1, 1), and (2, 0) on the triangle with vertices (0, 1), (2, -1), and (0, -3).
G.5.D Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.	Although the expectation only addresses two-dimensional figures, classroom activities can easily extend to three-dimensional figures. Students can also describe the symmetries, reflections across a plane, and rotations about a line for three-dimensional figures.

G.6. Additional Key Content

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.6.A Derive and apply formulas for arc length and area of a sector of a circle.	Example: • Find the area and perimeter of the Reauleaux triangle below. The Reuleaux triangle is constructed with three arcs. The center of each arc is located at the vertex of an equilateral triangle. Each arc extends between the two opposite vertices of the equilateral triangle. The figure below is a Reuleaux triangle that circumscribes equilateral triangle ABC. ΔABC has side length of 5 inches. $\overset{\frown}{AB}$ has center C, $\overset{\frown}{BC}$ has center A, and $\overset{\frown}{CA}$ has center B, and all three arcs have the same radius equal to the length of the sides of the triangle.



Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
G.6.B Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.	Examples: • Use a piece of string to measure the distance between two points on a ball or globe; verify that the string lies on an arc of a great circle. • On a globe, show with examples why airlines use polar routes instead of flying due east from Seattle to Paris. • Show that the sum of the angles of a triangle on a sphere is greater than 180 degrees.
G.6.C Apply formulas for surface area and volume of three-dimensional figures to solve problems.	Problems include those that are purely mathematical as well as those that arise in applied contexts. Three-dimensional figures include right and oblique prisms, pyramids, cylinders, cones, spheres, and composite three-dimensional figures. Examples: • As Pam scooped ice cream into a cone, she began to formulate a geometry problem in her mind. If the ice cream was perfectly spherical with diameter 2.25" and sat on a geometric cone that also had diameter 2.25" and was 4.5" tall, would the cone hold all the ice cream as it melted (without her eating any of it)? She figured the melted ice cream would have the same volume as the unmelted ice cream. Find the solution to Pam's problem and justify your reasoning. • A rectangle is 5 inches by 10 inches. Find the volume of a cylinder that is generated by rotating the rectangle about the 10-inch side.
G.6.D Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.	The emphasis in high school should be on verifying the relationships between length, area, and volume and on making predictions using algebraic methods. Examples: • What happens to the volume of a rectangular prism if four parallel edges are doubled in length? • The ratio of a pair of corresponding sides in two similar triangles is 5:3. The area of the smaller triangle is 108 in^2. What is the area of the larger triangle?

Performance Expectation *Students are expected to:*		***Explanatory Comments and Examples***
G.6.E	Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	Example: • The U.S. Census Bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. One can say that the U.S. population is 3 hundred million (3×10^8) and be precise to one digit. Although the population had surpassed 3 hundred million by the end of that month, explain why 3×10^8 remained precise to one digit.
G.6.F	Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.	This performance expectation is intended to build on students' knowledge of proportional relationships. Students should understand the relationship between scale factors and their inverses as they relate to choices about when to multiply and when to divide in converting measurements. Derived units include those that measure speed, density, flow rates, population density, etc. Example: • A digital camera takes pictures that are 3.2 megabytes in size. If the pictures are stored on a 1-gigabyte card, how many pictures can be taken before the card is full?

Mathematics 1

M1.3. Core Content: Linear functions, equations, and relationships

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M1.3.H Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.	Examples: • Write an equation for the perpendicular bisector of a given line segment. • Determine the equation of a line through the points (5, 3) and (5, -2). • Prove that the slopes of perpendicular lines are negative inverses of each other.

M1.4. Core Content: Proportionality, similarity, and geometric reasoning

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M1.4.A Distinguish between inductive and deductive reasoning.	Students generate and test conjectures inductively and then prove (or disprove) their conclusions deductively. Example: • A student first hypothesizes that the sum of the angles of a triangle is 180 degrees and then proves this is true. When was the student using inductive reasoning? When was s/he using deductive reasoning? Justify your answers.
M1.4.B Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.	Example: • Using dynamic geometry software, decide if the following is a plausible conjecture: If two parallel lines are cut by a transversal, then alternate interior angles are equal.
M1.4.C Use deductive reasoning to prove that a valid geometric statement is true.	Valid proofs may be presented in paragraph, two-column, or flow-chart formats. Proof by contradiction is a form of deductive reasoning. Example: • Prove that if two parallel lines are cut by a transversal, then alternate interior angles are equal
M1.4.D Determine and prove triangle similarity.	Similarity in Mathematics 1 builds on proportionality concepts from middle school mathematics. Determining and proving triangle congruence and other properties of triangles are included in Mathematics 2. Students should identify necessary and sufficient conditions for similarity in triangles, and use these conditions in proofs. Example: • For a given ΔRST, prove that ΔXYZ, formed by joining the midpoints of the sides of ΔRST, is similar to ΔRST.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M1.4.E Know, prove, and apply theorems about parallel and perpendicular lines.	Students should be able to summarize and explain basic theorems. They are not expected to recite lists of theorems, but they should know the conclusion of a theorem when given its hypothesis. Examples: • Prove that a point on the perpendicular bisector of a line segment is equidistant from the ends of the line segment. • If each of two lines is perpendicular to a given line, what is the relationship between the two lines? How do you know?
M1.4.F Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.	Example: • Take two parallel lines *l* and *m*, with (distinct) points A and B on *l* and C and D on *m*. If $\overleftrightarrow{AC}$ intersects $\overleftrightarrow{BD}$ at point E, prove that $\Delta ABE \approx \Delta CDE$.
M1.4.G Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.	Constructions using circles and lines with dynamic geometry software (i.e., virtual compass and straightedge) are equivalent to paper and pencil constructions. Example: • Construct and mathematically justify the steps to: — Bisect a line segment. — Drop a perpendicular from a point to a line. — Construct a line through a point that is parallel to another line.

Mathematics 2

M2.3. Core Content: Conjectures and proofs

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M2.3.A Use deductive reasoning to prove that a valid geometric statement is true.	Valid proofs may be presented in paragraph, two-column, or flow-chart formats. Proof by contradiction is a form of deductive reasoning. Example: • Prove that the diagonals of a rhombus are perpendicular bisectors of each other.
M2.3.B Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.	Example: • Identify errors in reasoning in the following proof: Given $\angle ABC \cong \angle PRQ$, $\overline{AB} \cong \overline{PQ}$, and $\overline{BC} \cong \overline{QR}$, then $\Delta ABC \cong \Delta PQR$ by SAS.

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
M2.3.C Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.	Examples: • If *m* and *n* are odd integers, then the sum of *m* and *n* is an even integer. State the converse and determine whether it is valid. • If a quadrilateral is a rectangle, the diagonals have the same length. State the contrapositive and determine whether it is valid.
M2.3.D Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.	Students sketch *points* and *lines* (undefined terms) and define and sketch representations of other common terms. They use definitions and postulates as they prove theorems throughout geometry. In their work with theorems, they identify the hypothesis and the conclusion and explain the role of each. Students describe the consequences of changing assumptions or using different definitions for subsequent theorems and logical arguments. Example: • There are two definitions of trapezoid that can be found in books or on the web. A trapezoid is either — a quadrilateral with exactly one pair of parallel sides or — a quadrilateral with at least one pair of parallel sides. Write some theorems that are true when applied to one definition but not the other, and explain your answer.
M2.3.E Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.	Examples: • Prove that the sum of the angles of a triangle is 180°. • Prove and explain theorems about the incenter, circumcenter, orthocenter, and centroid. • The rural towns of Atwood, Bridgeville, and Carnegie are building a communications tower to serve the needs of all three towns. They want to position the tower so that the distance from each town to the tower is equal. Where should they locate the tower? How far will it be from each town?
M2.3.F Determine and prove triangle congruence and other properties of triangles.	Students extend their work with similarity in Mathematics 1 to proving theorems about congruence and other properties of triangles. Students should identify necessary and sufficient conditions for congruence in triangles, and use these conditions in proofs. Examples: • Prove that congruent triangles are similar. • Show that not all SSA triangles are congruent.

Performance Expectations	***Explanatory Comments and Examples***

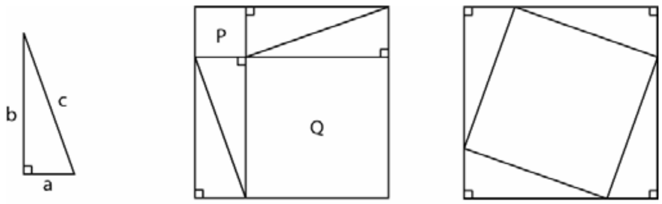
Students are expected to:

M2.3.G Know, prove, and apply the Pythagorean Theorem and its converse.

Students extend their work with the Pythagorean Theorem from previous grades to include formal proof.

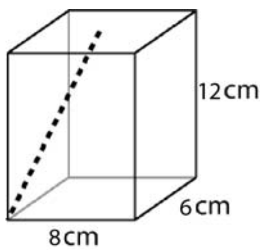
Examples:

- Consider any right triangle with legs *a* and *b* and hypotenuse *c*. The right triangle is used to create Figures 1 and 2. Explain how these figures constitute a visual representation of a proof of the Pythagorean Theorem.



Figures 1 Figure 2

- A juice box is 6 cm by 8 cm by 12 cm. A straw is inserted into a hole in the center of the top of the box. The straw must stick out 2 cm so you can drink from it. If the straw must be long enough to touch each bottom corner of the box, what is the minimum length the straw must be? (Assume the diameter of the straw is 0 for the mathematical model.)



- In $\triangle ABC$, with right angle at *C*, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.
- Apply the Pythagorean Theorem to derive the distance formula in the (*x*, *y*) plane.
- Determine the length of the altitude of an equilateral triangle whose side lengths measure 5 units.

M2.3.H Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.

Students apply their knowledge of the Pythagorean Theorem from Grade 8 to define the basic trigonometric ratios. They formally prove the Pythagorean Theorem in Mathematics 2.

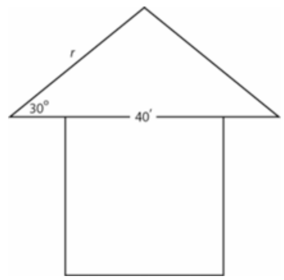
Examples:

- A 12-foot ladder leans against a wall to form a 63° angle with the ground. How many feet above the ground is the point on the wall at which the ladder is resting?
- Use the Pythagorean Theorem to establish that $\sin^2ø + \cos^2 ø = 1$ for ø between 0° and 90°.

Performance Expectations	*Explanatory Comments and Examples*
Students are expected to:	

M2.3.I Use the properties of special right triangles (30°–60°–90° and 45°–45°–90°) to solve problems.

Examples:

- If one leg of a right triangle has length 5 and the adjacent angle is 30°, what is the length of the other leg and the hypotenuse?
- If one leg of a 45°–45°–90° triangle has length 5, what is the length of the hypotenuse?
- The pitch of a symmetrical roof on a house 40 feet wide is 30º. What is the length of the rafter, *r*, exactly and approximately?
- 

M2.3.J Know, prove, and apply basic theorems about parallelograms.

Properties may include those that address symmetry and properties of angles, diagonals, and angle sums. Students may use inductive and deductive reasoning and counterexamples.

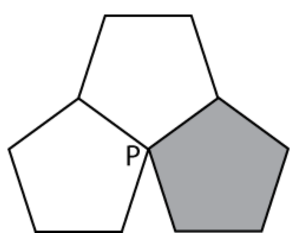
Examples:

- Are opposite sides of a parallelogram always congruent? Why or why not?
- Are opposite angles of a parallelogram always congruent? Why or why not?
- Prove that the diagonals of a parallelogram bisect each other.
- Explain why if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
- Prove that the diagonals of a rectangle are congruent. Is this true for any parallelogram? Justify your reasoning.

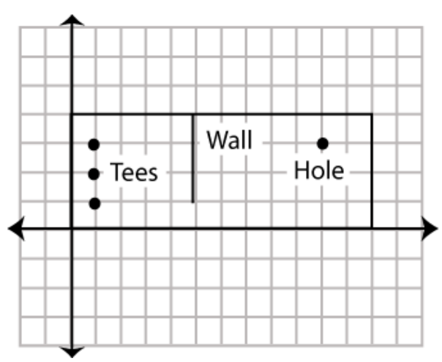
M2.3.K Know, prove, and apply theorems about properties of quadrilaterals and other polygons.

Examples:

- What is the length of the apothem of a regular hexagon with side length 8? What is the area of the hexagon?
- If the shaded pentagon were removed, it could be replaced by a regular *n*-sided polygon that would exactly fill the remaining space. Find the number of sides, *n*, of a replacement polygon that makes the three polygons fit perfectly.



Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M2.3.L Determine the coordinates of a point that is described geometrically.	Examples: • Determine the coordinates for the midpoint of a given line segment • Given the coordinates of three vertices of a parallelogram, determine all possible coordinates for the fourth vertex. • Given the coordinates for the vertices of a triangle, find the coordinates for the center of the circumscribed circle and the length of its radius.
M2.3.M Verify and apply properties of triangles and quadrilaterals in the coordinate plane.	Examples: • Given four points in a coordinate plane that are the vertices of a quadrilateral, determine whether the quadrilateral is a rhombus, a square, a rectangle, a parallelogram, or none of these. Name all the classifications that apply. • Given a parallelogram on a coordinate plane, verify that the diagonals bisect each other. • Given the line with *y*-intercept 4 and *x*-intercept 3, find the area of a square that has one corner on the origin and the opposite corner on the line described. • Below is a diagram of a miniature golf hole as drawn on a coordinate grid. The dimensions of the golf hole are 4 feet by 12 feet. Players must start their ball from one of the three tee positions, located at (1, 1), (1, 2), or (1, 3). The hole is located at (10, 3). A wall separates the tees from the hole. At which tee should the ball be placed to create the shortest "hole-in-one" path? Sketch the intended path of the ball, find the distance the ball will travel, and describe your reasoning. (Assume the diameters of the golf ball and the hole are 0 for the mathematical model.)



M2.5. Additional Key Content:

Performance Expectation *Students are expected to:*	**Explanatory Comments and Examples**
M2.5.B Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	Example: • The U.S. Census Bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. One can say that the U.S. population is 3 hundred million (3×10^8) and be precise to one digit. Although the population had surpassed 3 hundred million by the end of that month, explain why 3×10^8 remained precise to one digit.
M2.5.C Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.	This performance expectation is intended to build on students' knowledge of proportional relationships. Students should understand the relationship between scale factors and their inverses as they relate to choices about when to multiply and when to divide in converting measurements. Derived units include those that measure speed, density, flow rates, population density, etc. Example: • A digital camera takes pictures that are 3.2 megabytes in size. If the pictures are stored on a 1-gigabyte card, how many pictures can be taken before the card is full?

Mathematics 3

M3.2. Core Content: Transformations and functions

Performance Expectation *Students are expected to:*	Explanatory Comments and Examples
M3.2.A Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.	Transformations include translations, rotations, reflections, and dilations. Example: • Line *m* is described by the equation $y = 2x + 3$. Graph line *m* and reflect line *m* across the line $y = x$. Determine the equation of the image of the reflection. Describe the relationship between the line and its image.
M3.2.B Determine and apply properties of transformations.	Students make and test conjectures about compositions of transformations and inverses of transformations, the commutativity and associativity of transformations, and the congruence and similarity of two-dimensional figures under various transformations. Examples: • Identify transformations (alone or in composition) that preserve congruence. • Determine whether the composition of two reflections of a line is commutative. • Determine whether the composition of two rotations about the same point of rotation is commutative. • Find a rotation that is equivalent to the composition of two reflections over intersecting lines. • Find the inverse of a given transformation.
M3.2.C Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.	Examples: • Find a sequence of transformations that superimposes the segment with endpoints (0, 0) and (2, 1) on the segment with endpoints (4, 2) and (3, 0). • Find a sequence of transformations that superimposes the triangle with vertices (0, 0), (1, 1), and (2, 0) on the triangle with vertices (0, 1), (2, -1), and (0, -3).
M3.2.D Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.	Although the expectation only addresses two-dimensional figures, classroom activities can easily extend to three-dimensional figures. Students can also describe the symmetries, reflections across a plane, and rotations about a line for three-dimensional figures.

M3.5. Core Content: Three-dimensional geometry

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M3.5.A Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.	Example: • Describe all the ways that three planes can intersect in space.
M3.5.B Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.	Examples: • Given the number of faces of a regular polyhedron, derive a formula for the number of vertices. • Describe symmetries of three-dimensional polyhedra and their two-dimensional faces. • Describe the lateral faces that are required for a pyramid to be a right pyramid with a regular base. Describe the lateral faces required for an oblique pyramid that has a regular base.
M3.5.C Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.	Examples: • Start with a regular tetrahedron with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the tetrahedron is a square. Find the area of the square. (Requires no pencil or paper.) • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a regular hexagon. Find the area of the hexagon. • Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a rectangle that is not a face and contains four of the vertices. Find the area of the rectangle. • Which has the larger area, the above rectangle or the above hexagon?
M3.5.D Apply formulas for surface area and volume of three-dimensional figures to solve problems.	Problems include those that are purely mathematical as well as those that arise in applied contexts. Three-dimensional figures include right and oblique prisms, pyramids, cylinders, cones, spheres, and composite three-dimensional figures. Examples: • As Pam scooped ice cream into a cone, she began to formulate a geometry problem in her mind. If the ice cream was perfectly spherical with diameter 2.25" and sat on a geometric cone that also had diameter 2.25" and was 4.5" tall, would the cone hold all the ice cream as it melted (without her eating any of it)? She figured the melted ice cream would have the same volume as the unmelted ice cream. Find the solution to Pam's problem and justify your reasoning. • A rectangle is 5 inches by 10 inches. Find the volume of a cylinder that is generated by rotating the rectangle about the 10-inch side.

Performance Expectations Students are expected to:	Explanatory Comments and Examples
M3.5.E Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.	The emphasis in high school should be on verifying the relationships between length, area, and volume and on making predictions using algebraic methods. Examples: • What happens to the volume of a rectangular prism if four parallel edges are doubled in length? • The ratio of a pair of corresponding sides in two similar triangles is 5:3. The area of the smaller triangle is 108 in^2. What is the area of the larger triangle?
M3.5.F Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.	Examples: • Use a piece of string to measure the distance between two points on a ball or globe; verify that the string lies on an arc of a great circle. • On a globe, show with examples why airlines use polar routes instead of flying due east from Seattle to Paris. • Show that the sum of the angles of a triangle on a sphere is greater than 180 degrees.

M3.7. Additional Key Content:

Performance Expectation Students are expected to:	Explanatory Comments and Examples
M3.7.A Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.	Examples: • Given a line tangent to a circle, know and explain that the line is perpendicular to the radius drawn to the point of tangency. • Prove that two chords equally distant from the center of a circle are congruent. • Prove that if one side of a triangle inscribed in a circle is a diameter, then the triangle is a right triangle. • Prove that if a radius of a circle is perpendicular to a chord of a circle, then the radius bisects the chord.
M3.7.B Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).	Examples: • Write an equation for a circle with a radius of 2 units and center at (1, 3). • Given the circle $x^2 + y^2 = 4$ and the line $y = x$, find the points of intersection. • Write an equation for a circle given a line segment as a diameter. • Write an equation for a circle determined by a given center and tangent line.

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.7.C Explain and perform constructions related to the circle.	Students perform constructions using straightedge and compass, paper folding, and dynamic geometry software. What is important is that students understand the mathematics and are able to justify each step in a construction.

Example:

- In each case, explain why the constructions work:
    a. Construct the center of a circle from two chords.
    b. Construct a circumscribed circle for a triangle.
    c. Inscribe a circle in a triangle.

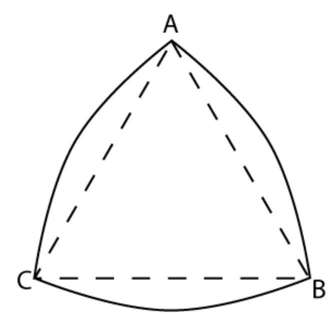
M3.7.D Derive and apply formulas for arc length and area of a sector of a circle.

Example:

- Find the area and perimeter of the Reauleaux triangle below.

  The Reuleaux triangle is constructed with three arcs. The center of each arc is located at the vertex of an equilateral triangle. Each arc extends between the two opposite vertices of the equilateral triangle.

  The figure below is a Reuleaux triangle that circumscribes equilateral triangle ABC. $\triangle ABC$ has side length of 5 inches. $\overset{\frown}{AB}$ has center C, $\overset{\frown}{BC}$ has center A, and $\overset{\frown}{CA}$ has center B, and all three arcs have the same radius equal to the length of the sides of the triangle.



Data Analysis, Statistics, and Probability Strand

In kindergarten through grade 5, students learn a variety of ways to display data, and they interpret data to answer questions. Students in grades 3–5 relate what they are learning about fractions to describe the likelihood of something happening. In grades 6–8, students use a more extensive set of tools to summarize and analyze data sets, and they extend their work with probability. **In grades 9–12, students use more sophisticated tools to represent, describe, and compare data sets, and they analyze statistical studies and findings to determine whether conclusions are sound.**

This is one of six strand documents that accompany the Washington State K–8 Mathematics Standards, tracking the development of important mathematical ideas and skills across grades K–8. Where content of an expectation may address more than one strand, that expectation may appear in more than one strand document.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***

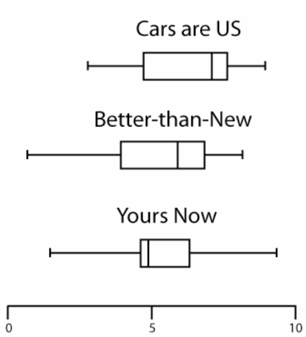
A1.6.A Use and evaluate the accuracy of summary statistics to describe and compare data sets.

A univariate set of data identifies data on a single variable, such as shoe size.

This expectation extends what students have learned in earlier grades to include evaluation and justification. They both compute and evaluate the appropriateness of measure of center and spread (range and interquartile range) and use these measures to accurately compare data sets. Students will draw appropriate conclusions through the use of statistical measures of center, frequency, and spread, combined with graphical displays.

Examples:

- The local minor league baseball team has a salary dispute. Players claim they are being underpaid, but managers disagree.
  - — Bearing in mind that a few top players earn salaries that are quite high, would it be in the managers' best interest to use the mean or median when quoting the "average" salary of the team? Why?
  - — What would be in the players' best interest?
- Each box-and-whisker plot shows the prices of used cars (in thousands of dollars) advertised for sale at three different car dealers. If you want to go to the dealer whose prices seem least expensive, which dealer would you go to? Use statistics from the displays to justify your answer.



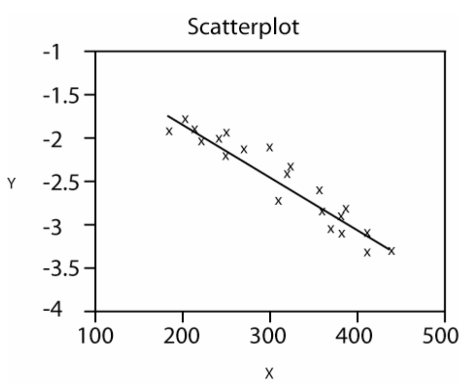
A1.6.B Make valid inferences and draw conclusions based on data.

Determine whether arguments based on data confuse association with causation. Evaluate the reasonableness of and make judgments about statistical claims, reports, studies, and conclusions.

Example:

- Mr. Shapiro found that the amount of time his students spent doing mathematics homework is positively correlated with test grades in his class. He concluded that doing homework makes students' test scores higher. Is this conclusion justified? Explain any flaws in Mr. Shapiro's reasoning.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A1.6.C Describe how linear transformations affect the center and spread of univariate data.	Examples: • A company decides to give every one of its employees a $5,000 raise. What happens to the mean and standard deviation of the salaries as a result? • A company decides to double each of its employee's salaries. What happens to the mean and standard deviation of the salaries as a result?
A1.6.D Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and *y*-intercept of the line, and use the equation to make predictions.	A bivariate set of data presents data on two variables, such as shoe size and height. In high school, the emphasis is on using a line of best fit to interpret data and on students making judgments about whether a bivariate data set can be modeled with a linear function. Students can use various methods, including technology, to obtain a line of best fit. Making predictions involves both interpolating and extrapolating from the original data set. Students need to be able to evaluate the quality of their predictions, recognizing that extrapolation is based on the assumption that the trend indicated continues beyond the unknown data.
A1.6.E Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.	Example: • Which words—*strong* or *weak*, *positive* or *negative*—could be used to describe the correlation shown in the sample scatterplot below?



Algebra 2

A2.1. Core Content: Solving problems

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.1.F Solve problems involving combinations and permutations.	Examples: • The company Ali works for allows her to invest in her choice of 10 different mutual funds, 6 of which grew by at least 5% over the last year. Ali randomly selected 4 of the 10 funds in which to invest. What is the probability that 3 of Ali's funds grew by 5%? • Four points (A, B, C, and D) lie on one straight line, n, and five points (E, F, G, H, and J) lie on another straight line, m, that is parallel to n. What is the probability that three points, selected at random, will form a triangle?

A2.6. Core Content: Probability, data, and dstributions

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.6.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).	
A2.6.B Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.	Example: • What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced?
A2.6.C Compute permutations and combinations, and use the results to calculate probabilities.	Example: • Two friends, Abby and Ben, are among five students being considered for three student council positions. If each of the five students has an equal likelihood of being selected, what is the probability that Abby and Ben will both be selected?

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.6.D Apply the binomial theorem to solve problems involving probability.	The binominal theorem is also applied when computing with polynomials. Examples: • Use Pascal's triangle and the binomial theorem to find the number of ways six objects can be selected four at a time. • In a survey, 33% of adults reported that they preferred to get the news from newspapers rather than television. If you survey 5 people, what is the probability of getting exactly 2 people who say they prefer news from the newspaper? — Write an equation that can be used to solve the problem. — Create a histogram of the binomial distribution of the probability of getting 0 through 5 responders saying they prefer the newspaper.
A2.6.E Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	In high school, determining a formula for a curve of best fit requires a graphing calculator or similar technological tool.
A2.6.F Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.	Students should be able to identify unimodality, symmetry, standard deviation, spread, and the shape of a data curve to determine whether the curve could reasonably be approximated by a normal distribution. Given formulas, student should be able to calculate the standard deviation for a small data set, but calculators ought to be used if there are very many points in the data set. It is important that students be able to describe the characteristics of the normal distribution and identify common examples of data that are and are not reasonably modeled by it. Common examples of distributions that are approximately normal include physical performance measurements (e.g., weightlifting, timed runs), heights, and weights. Apply the Empirical Rule (68–95–99.7 Rule) to approximate the percentage of the population meeting certain criteria in a normal distribution. Example: • Which is more likely to be affected by an outlier in a set of data, the interquartile range or the standard deviation?

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
A2.6.G Calculate and interpret margin of error and confidence intervals for population proportions.	Students will use technology based on the complexity of the situation.

Students use confidence intervals to critique various methods of statistical experimental design, data collection, and data presentation used to investigate important problems, including those reported in public studies.

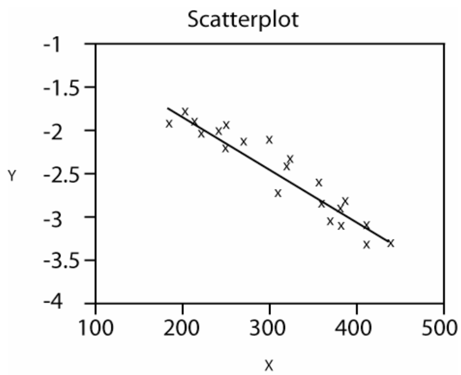
Example:

- In 2007, 400 of the 500 10th graders in Local High School passed the WASL. In 2008, 375 of the 480 10th graders passed the test. The Local Gazette headline read "10th Grade WASL Scores Decline in 2008!" In response, the Superintendent of Local School District wrote a letter to the editor claiming that, in fact, WASL performance was not significantly lower in 2008 than it was in 2007. Who is correct, the Local Gazette or the Superintendent?

  Use mathematics to find the margin of error to justify your conclusion. (Formula for the margin of error (E): $E = z_c\sqrt{\frac{p(1-p)}{n}}$; $z_{95} = 1.96$, where n is the sample size, p is the proportion of the sample with the trait of interest, c is the confidence level, and z_c is the multiplier for the specified confidence interval.)

Mathematics 1

M1.3. Core Content: Linear functions, equations, and relationships

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M1.3.G Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.	Example: • Which words—strong or weak, positive or negative—could be used to describe the correlation shown in the sample scatterplot below? 
M1.3.H Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.	Examples: • Write an equation for the perpendicular bisector of a given line segment. • Determine the equation of a line through the points (5, 3) and (5, -2). • Prove that the slopes of perpendicular lines are negative inverses of each other.

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***

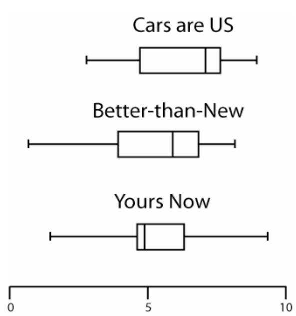
M1.5.A Use and evaluate the accuracy of summary statistics to describe and compare data sets.

A univariate set of data identifies data on a single variable, such as shoe size.

This expectation extends what students have learned in earlier grades to include evaluation and justification. They both compute and evaluate the appropriateness of measure of center and spread (range and interquartile range) and use these measures to accurately compare data sets. Students will draw appropriate conclusions through the use of statistical measures of center, frequency, and spread, combined with graphical displays.

Examples:

- The local minor league baseball team has a salary dispute. Players claim they are being underpaid, but managers disagree.
  - — Bearing in mind that a few top players earn salaries that are quite high, would it be in the managers' best interest to use the mean or median when quoting the "average" salary of the team? Why?
  - — What would be in the players' best interest?
- Each box-and-whisker plot shows the prices of used cars (in thousands of dollars) advertised for sale at three different car dealers. If you want to go to the dealer whose prices seem least expensive, which dealer would you go to? Use statistics from the displays to justify your answer.



M1.5.B Describe how linear transformations affect the center and spread of univariate data.

Examples:

- A company decides to give every one of its employees a $5,000 raise. What happens to the mean and standard deviation of the salaries as a result?
- A company decides to double each of its employee's salaries. What happens to the mean and standard deviation of the salaries as a result?

M1.5.C Make valid inferences and draw conclusions based on data.

Determine whether arguments based on data confuse association with causation. Evaluate the reasonableness of and make judgments about statistical claims, reports, studies, and conclusions

Example:

- Mr. Shapiro found that the amount of time his students spent doing mathematics homework is positively correlated with test grades in his class. He concluded that doing homework makes students' test scores higher. Is this conclusion justified? Explain any flaws in Mr. Shapiro's reasoning.

Mathematics 2

M2.2. Core Content: Quadratic functions, equations, and relationships

Performance Expectation *Students are expected to:*	***Explanatory Comments and Examples***
M2.1.E Solve problems involving combinations and permutations.	Examples: • The company Ali works for allows her to invest in her choice of 10 different mutual funds, 6 of which grew by at least 5% over the last year. Ali randomly selected 4 of the 10 funds in which to invest. What is the probability that 3 of Ali's funds grew by 5%? • Four points (A, B, C, and D) lie on one straight line, n, and five points (E, F, G, H, and J) lie on another straight line, m, that is parallel to n. What is the probability that three points, selected at random, will form a triangle?
M2.2.H Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.	In high school, determining a formula for a curve of best fit requires a graphing calculator or similar technological tool.
M2.4.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).	
M2.4.B Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.	Example: • What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced?
M2.4.C Compute permutations and combinations, and use the results to calculate probabilities.	Example: • Two friends, Abby and Ben, are among five students being considered for three student council positions. If each of the five students has an equal likelihood of being selected, what is the probability that Abby and Ben will both be selected?
M2.4.D Apply the binomial theorem to solve problems involving probability.	The binominal theorem is also applied when computing with polynomials. Examples: • Use Pascal's triangle and the binomial theorem to find the number of ways six objects can be selected four at a time. • In a survey, 33% of adults reported that they preferred to get the news from newspapers rather than television. If you survey 5 people, what is the probability of getting exactly 2 people who say they prefer news from the newspaper? Write an equation that can be used to solve the problem. Create a histogram of the binomial distribution of the probability of getting 0 through 5 responders saying they prefer the newspaper.

Mathematics 3

M3.4. Core Content: Quantifying variability

Performance Expectations *Students are expected to:*	***Explanatory Comments and Examples***
M3.4.A Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.	Students should be able to identify unimodality, symmetry, standard deviation, spread, and the shape of a data curve to determine whether the curve could reasonably be approximated by a normal distribution. Given formulas, student should be able to calculate the standard deviation for a small data set, but calculators ought to be used if there are very many points in the data set. It is important that students be able to describe the characteristics of the normal distribution and identify common examples of data that are and are not reasonably modeled by it. Common examples of distributions that are approximately normal include physical performance measurements (e.g., weightlifting, timed runs), heights, and weights. Apply the Empirical Rule (68–95–99.7 Rule) to approximate the percentage of the population meeting certain criteria in a normal distribution. Example: • Which is more likely to be affected by an outlier in a set of data, the interquartile range or the standard deviation?
M3.4.B Calculate and interpret margin of error and confidence intervals for population proportions.	Students will use technology based on the complexity of the situation. Students use confidence intervals to critique various methods of statistical experimental design, data collection, and data presentation used to investigate important problems, including those reported in public studies. Example: • In 2007, 400 of the 500 10th graders in Local High School passed the WASL. In 2008, 375 of the 480 10th graders passed the test. The Local Gazette headline read "10th Grade WASL Scores Decline in 2008!" In response, the Superintendent of Local School District wrote a letter to the editor claiming that, in fact, WASL performance was not significantly lower in 2008 than it was in 2007. Who is correct, the Local Gazette or the Superintendent? Use mathematics to find the margin of error to justify your conclusion. (Formula for the margin of error (E): $E = z_c\sqrt{\frac{p(1-p)}{n}}$; $z_{95} = 1.96$, where n is the sample size, p is the proportion of the sample with the trait of interest, c is the confidence level, and z_c is the multiplier for the specified confidence interval.)