

A1.7. Additional Key Content (**Algebra**)

Students develop a basic understanding of arithmetic and geometric sequences and of exponential functions, including their graphs and other representations. They use exponential functions to analyze relationships, represent and model problems, and answer questions in situations that are modeled by these nonlinear functions. Students learn graphical and numerical methods for approximating solutions to exponential equations. Students interpret the meaning of problem solutions and explain limitations related to solutions.

A1.8. Core Processes: **Reasoning, problem solving, and communication**

Students formalize the development of reasoning in Algebra 1 as they use algebra and the properties of number systems to develop valid mathematical arguments, make and prove conjectures, and find counterexamples to refute false statements, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

**Mathematics content based on
Adopted Washington State
High School Mathematics Standards,
August 22, 2008,
Office of the Superintendent of Public
Instruction**

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Adopted Washington State Mathematics Standards

Algebra 1 August 22, 2008

A1.1. Core Content: **Solving problems (Algebra)**

Students learn to solve many new types of problems in Algebra 1, and this first core content area highlights the types of problems students will be able to solve after they master the concepts and skills in this course. Students are introduced to several types of functions, including exponential and functions defined piecewise, and they spend considerable time with linear and quadratic functions. Each type of function included in Algebra 1 provides students a tool to solve yet another class of problems. They learn that specific functions model situations described in word problems, and so functions are used to solve various types of problems. The ability to determine functions and write equations that represent problems is an important mathematical skill in itself. Many problems that initially appear to be very different from each other can actually be represented by identical equations. Students encounter this important and unifying principle of algebra—that the same algebraic techniques can be applied to a wide variety of different situations.

A1.2. Core Content: **Numbers, expressions, and operations (Numbers, Operations, Algebra)**

Students see the number system extended to the real numbers represented by the number line. They work with integer exponents, scientific notation, and radicals, and use variables and expressions to solve problems from purely mathematical as well as applied contexts. They build on their understanding of computation using arithmetic operations and properties and expand this understanding to include the symbolic language of algebra. Students demonstrate this ability to write and manipulate a wide variety of algebraic expressions throughout high school mathematics as they apply algebraic procedures to solve problems.

A1.3. Core Content: **Characteristics and behaviors of functions (Algebra)**

Students formalize and deepen their understanding of functions, the defining characteristics and uses of functions, and the mathematical language used to describe functions. They learn that functions are often specified by an equation of the form $y = f(x)$,

where any allowable x-value yields a unique y-value. While Algebra 1 has a particular focus on linear and quadratic equations and systems of equations, students also learn about exponential functions and those that can be defined piecewise, particularly step functions and functions that contain the absolute value of an expression. Students learn about the representations and basic transformations of these functions and the practical and mathematical limitations that must be considered when working with functions and when using functions to model situations.

A1.4. Core Content: **Linear functions, equations, and inequalities (Algebra)**

Students understand that linear functions can be used to model situations involving a constant rate of change. They build on the work done in middle school to solve sets of linear equations and inequalities in two variables, learning to interpret the intersection of the lines as the solution. While the focus is on solving equations, students also learn graphical and numerical methods for approximating solutions to equations. They use linear functions to analyze relationships, represent and model problems, and answer questions. These algebraic skills are applied in other Core Content areas across high school courses.

A1.5. Core Content: **Quadratic functions and equations (Algebra)**

Students study quadratic functions and their graphs, and solve quadratic equations with real roots in Algebra 1. They use quadratic functions to represent and model problems and answer questions in situations that are modeled by these functions. Students solve quadratic equations by factoring and computing with polynomials. The important mathematical technique of completing the square is developed enough so that the quadratic formula can be derived.

A1.6. Core Content: **Data and distributions (Data/Statistics/Probability)**

Students select mathematical models for data sets and use those models to represent, describe, and compare data sets. They analyze data to determine the relationship between two variables and make and defend appropriate predictions, conjectures, and generalizations. Students understand limitations of conclusions based on results of a study or experiment and recognize common misconceptions and misrepresentations in interpreting conclusions.

Algebra 1 Performance Expectations

Solving Problems (algebra)

- A1.1.A Select and justify functions and equations to model and solve problems.
- A1.1.B Solve problems that can be represented by linear functions, equations, and inequalities.
- A1.1.C Solve problems that can be represented by a system of two linear equations or inequalities.
- A1.1.D Solve problems that can be represented by quadratic functions and equations.
- A1.1.E Solve problems that can be represented by exponential functions and equations.

Numbers, Expressions, and Operations (numbers, operations, algebra)

- A1.2.A Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.
- A1.2.B Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.
- A1.2.C Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.
- A1.2.D Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.
- A1.2.E Use algebraic properties to factor and combine like terms in polynomials.
- A1.2.F Add, subtract, multiply, and divide polynomials.

Characteristics and Behaviors of Functions (algebra)

- A1.3.A Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.
- A1.3.B Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.
- A1.3.C Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.

Linear Functions, Equations, and Inequalities (algebra)

- A1.4.A Write and solve linear equations and inequalities in one variable.
- A1.4.B Write and graph an equation for a line given the slope and the y -intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.
- A1.4.C Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.
- A1.4.D Write and solve systems of two linear equations and inequalities in two variables.
- A1.4.E Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.

Quadratic Functions and Equations (algebra)

- A1.5.A Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.
- A1.5.B Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the x -intercepts as solutions to a quadratic equation.
- A1.5.C Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where a , b , c , and d are integers.
- A1.5.D Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.

Data and Distributions (data/statistics/probability)

- A1.6.A Use and evaluate the accuracy of summary statistics to describe and compare data sets.
- A1.6.B Make valid inferences and draw conclusions based on data.
- A1.6.C Describe how linear transformations affect the center and spread of univariate data.
- A1.6.D Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and y -intercept of the line, and use the equation to make predictions.
- A1.6.E Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.

Additional Key Content (algebra)

- A1.7.A Sketch the graph for an exponential function of the form $y = ab^n$ where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.
- A1.7.B Find and approximate solutions to exponential equations.
- A1.7.C Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.
- A1.7.D Solve an equation involving several variables by expressing one variable in terms of the others.

Reasoning, Problem Solving, and Communication

- A1.8.A Analyze a problem situation and represent it mathematically.
- A1.8.B Select and apply strategies to solve problems.
- A1.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- A1.8.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.
- A1.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- A1.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
- A1.8.G Synthesize information to draw conclusions, and evaluate the arguments and conclusions of others.
- A1.8.H Use inductive reasoning about algebra and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.

correct mathematical language, terms, symbols, and conventions as they address problems in Geometry and provide descriptions and justifications of solution processes. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways, and more and more occupations and fields of study rely on mathematics.

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Geometry

August 22, 2008

G.1. Core Content: *Logical arguments and proofs (Logic)*

Students formalize the reasoning skills they have developed in previous grades and solidify their understanding of what it means to prove a geometric statement mathematically. In Geometry, students encounter the concept of formal proof built on definitions, axioms, and theorems. They use inductive reasoning to test conjectures about geometric relationships and use deductive reasoning to prove or disprove their conclusions. Students defend their reasoning using precise mathematical language and symbols.

G.2 Core Content: *Lines and angles (Geometry/Measurement)*

Students study basic properties of parallel and perpendicular lines, their respective slopes, and the properties of the angles formed when parallel lines are intersected by a transversal. They prove related theorems and apply them to solve both mathematical and practical problems.

G.3. Core Content: *Two- and three-dimensional figures (Geometry/Measurement)*

Students know and can prove theorems about two- and three-dimensional geometric figures, both formally and informally. They identify necessary and sufficient conditions for proving congruence, similarity, and properties of figures. Triangles are a primary focus, beginning with general properties of triangles, working with right triangles and special triangles, proving and applying the Pythagorean Theorem and its converse, and applying the basic trigonometric ratios of sine, cosine, and tangent. Students extend their learning to other polygons and the circle, and do some work with three-dimensional figures.

G.4. Core Content: *Geometry in the coordinate plane (Geometry/Measurement, Algebra)*

Students make connections between geometry and algebra by studying geometric properties and

attributes that can be represented on the coordinate plane. They use the coordinate plane to represent situations that are both purely mathematical and that arise in applied contexts. In this way, they use the power of algebra to solve problems about shapes and space.

G.5. Core Content: *Geometric transformations (Geometry/Measurement)*

Students continue their study of geometric transformations, focusing on the effect of such transformations and the composition of transformations on the attributes of geometric figures. They study techniques for establishing congruence and similarity by means of transformations.

G.6. Additional Key Content (*Measurement*)

Students extend and formalize their work with geometric formulas for perimeter, area, surface area, and volume of two- and three-dimensional figures, focusing on mathematical derivations of these formulas and their applications in complex problems. They use properties of geometry and measurement to solve problems in purely mathematical as well as applied contexts. Students understand the role of units in measurement and apply what they know to solve problems involving derived measures like speed or density. They understand that all measurement is approximate and specify precision in measurement problems.

G.7. Core Processes: *Reasoning, problem solving, and communication*

Students formalize the development of reasoning in Geometry as they become more sophisticated in their ability to reason inductively and begin to use deductive reasoning in formal proofs. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students use a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. They use

Geometry Performance Expectations

Logical Arguments and Proofs (logic)

- G.1.A Distinguish between inductive and deductive reasoning.
- G.1.B Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.
- G.1.C Use deductive reasoning to prove that a valid geometric statement is true.
- G.1.D Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.
- G.1.E Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.
- G.1.F Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.

Lines and Angles (geometry/measurement)

- G.2.A Know, prove, and apply theorems about parallel and perpendicular lines.
- G.2.B Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.
- G.2.C Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.
- G.2.D Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.

Two- and Three-Dimensional Figures (geometry/measurement)

- G.3.A Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.
- G.3.B Determine and prove triangle congruence, triangle similarity, and other properties of triangles.
- G.3.C Use the properties of special right triangles ($30^\circ-60^\circ-90^\circ$ and $45^\circ-45^\circ-90^\circ$) to solve problems.
- G.3.D Know, prove, and apply the Pythagorean Theorem and its converse.
- G.3.E Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.
- G.3.F Know, prove, and apply basic theorems about parallelograms.
- G.3.G Know, prove, and apply theorems about properties of quadrilaterals and other polygons.
- G.3.H Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.
- G.3.I Explain and perform constructions related to the circle.
- G.3.J Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.
- G.3.K Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.

Geometry in the Coordinate Plane (geometry/measurement, algebra)

- G.4.A Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.
- G.4.B Determine the coordinates of a point that is described geometrically.
- G.4.C Verify and apply properties of triangles and quadrilaterals in the coordinate plane.
- G.4.D Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).

Geometric Transformations (geometry/measurement)

- G.5.A Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.
- G.5.B Determine and apply properties of transformations.
- G.5.C Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.
- G.5.D Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.

Additional Key Content (measurement)

- G.6.A Derive and apply formulas for arc length and area of a sector of a circle.
- G.6.B Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.
- G.6.C Apply formulas for surface area and volume of three-dimensional figures to solve problems.
- G.6.D Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.
- G.6.E Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.
- G.6.F Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.

Reasoning, Problem Solving, and Communication

- G.7.A Analyze a problem situation and represent it mathematically.
- G.7.B Select and apply strategies to solve problems.
- G.7.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- G.7.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.
- G.7.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- G.7.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
- G.7.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.
- G.7.H Use inductive reasoning to make conjectures, and use deductive reasoning

and partial and infinite sums of geometric series. This conceptual understanding of series lays an important foundation for understanding calculus.

A2.8. Core Processes: Reasoning, problem solving, and communication

Students formalize the development of reasoning at high school as they use algebra and the properties of number systems to develop valid mathematical arguments, make and prove conjectures, and find counterexamples to refute false statements using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

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Algebra 2 August 22, 2008

A2.1. Core Content: Solving problems

The first core content area highlights the type of problems students will be able to solve by the end of Algebra 2, as they extend their ability to solve problems with additional functions and equations. When presented with a word problem, students are able to determine which function or equation models the problem and use that information to solve the problem. They build on what they learned in Algebra 1 about linear and quadratic functions and are able to solve more complex problems. Additionally, students learn to solve problems modeled by exponential and logarithmic functions, systems of equations and inequalities, inverse variations, and combinations and permutations. Turning word problems into equations that can be solved is a skill students hone throughout Algebra 2 and subsequent mathematics courses.

A2.2. Core Content: Numbers, expressions, and operations (Numbers, Operations, Algebra)

Students extend their understanding of number systems to include complex numbers, which they will see as solutions for quadratic equations. They grow more proficient in their use of algebraic techniques as they continue to use variables and expressions to solve problems. As problems become more sophisticated and the level of mathematics increases, so does the complexity of the symbolic manipulations and computations necessary to solve the problems. Students refine the foundational algebraic skills they need to be successful in subsequent mathematics courses.

A2.3. Core Content: Quadratic functions and equations (Algebra)

As students continue to solve quadratic equations and inequalities in Algebra 2, they encounter complex roots for the first time. They learn to translate between forms of quadratic equations, applying the vertex form to evaluate maximum and minimum values and find symmetry of the graph, and they learn to identify which form should be used in a particular situation. This opens up a whole range of new problems students can solve using quadratics. These algebraic skills are applied in subsequent high school mathematics and statistics courses.

A2.4. Core Content: Exponential and logarithmic functions and equations (Algebra)

Students extend their understanding of exponential functions from Algebra 1 with an emphasis on inverse functions. This leads to a natural introduction of logarithms and logarithmic functions. They learn to use the basic properties of exponential and logarithmic functions, graphing both types of function to analyze relationships, represent and model problems, and answer questions. Students employ these functions in many practical situations, such as applying exponential functions to determine compound interest and applying logarithmic functions to determine the pH of a liquid.

A2.5. Core Content: Additional functions and equations (Algebra)

Students learn about additional classes of functions including square root, cubic, logarithmic, and those involving inverse variation. Students plot points and sketch graphs to represent these functions and use algebraic techniques to solve related equations. In addition to studying the defining characteristics of each of these classes of functions, students gain the ability to construct new functions algebraically and using transformations. These extended skills and techniques serve as the foundation for further study and analysis of functions in subsequent mathematics courses.

A2.6. Core Content: Probability, data, and distributions (Data/Statistics/Probability)

Students formalize their study of probability, computing both combinations and permutations to calculate the likelihood of an outcome in uncertain circumstances and applying the binomial theorem to solve problems. They extend their use of statistics to graph bivariate data and analyze its shape to make predictions. They calculate and interpret measures of variability, confidence intervals, and margins of error for population proportions. Dual goals underlie the content in the section: students prepare for the further study of statistics and become thoughtful consumers of data.

A2.7. Additional Key Content (Algebra)

Students study two important topics here. First, they extend their ability to solve systems of two equations in two variables to solving systems of three equations in three variables, which leads to the full development of matrices in Precalculus. Second, they formalize their work with series as they learn to find the terms and partial sums of arithmetic series and the terms

Algebra 2 Performance Expectations

Solving Problems

- A2.1.A Select and justify functions and equations to model and solve problems.
- A2.1.B Solve problems that can be represented by systems of equations and inequalities.
- A2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.
- A2.1.D Solve problems that can be represented by exponential and logarithmic functions and equations.
- A2.1.E Solve problems that can be represented by inverse variations of the forms:
 $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx+c)}$.
- A2.1.F Solve problems involving combinations and permutations.

Numbers, Expressions, and Operations (numbers, operations, algebra)

- A2.2.A Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.
- A2.2.B Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.
- A2.2.C Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.

Quadratic Functions and Equations (algebra)

- A2.3.A Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.
- A2.3.B Determine the number and nature of the roots of a quadratic function.
- A2.3.C Solve quadratic equations and inequalities, including equations with complex roots.

Exponential and Logarithmic Functions and Equations (algebra)

- A2.4.A Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.
- A2.4.B Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.
- A2.4.C Solve exponential and logarithmic equations.

Additional Functions and Equations (algebra)

- A2.5.A Construct new functions using the transformations $f(x-h)$, $f(x)+k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).
- A2.5.B Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x-c} + d$, and solve related equations.
- A2.5.C Plot points, sketch, and describe the graphs of functions of the forms:
 $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx+c)}$, and solve related equations.
- A2.5.D Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.

Probability, Data, and Distributions (data/statistics/probability)

- A2.6.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).
- A2.6.B Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.
- A2.6.C Compute permutations and combinations, and use the results to calculate probabilities.
- A2.6.D Apply the binomial theorem to solve problems involving probability.
- A2.6.E Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.
- A2.6.F Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.
- A2.6.G Calculate and interpret margin of error and confidence intervals for population proportions.

Additional Key Content (algebra)

- A2.7.A Solve systems of three equations with three variables.
- A2.7.B Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.

Reasoning, Problem Solving, and Communication

- A2.8.A Analyze a problem situation and represent it mathematically.
- A2.8.B Select and apply strategies to solve problems.
- A2.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- A2.8.D Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.
- A2.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- A2.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
- A2.8.G Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.
- A2.8.H Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.

use variables and expressions to solve problems from purely mathematical as well as applied contexts. They build on their understanding of and ability to compute with arithmetic operations and properties and expand this understanding to include the symbolic language of algebra. Students demonstrate this ability to write and manipulate a wide variety of algebraic expressions throughout high school mathematics as they apply algebraic procedures to solve problems.

M1.7. Additional Key Content (Numbers, Algebra)
Students develop a basic understanding of arithmetic and geometric sequences and of exponential functions, including their graphs and other representations. They use exponential functions to analyze relationships, represent and model problems, and answer questions in situations that are modeled by these nonlinear functions. Students learn graphical and numerical methods for approximating solutions to exponential equations. Students interpret the meaning of problem solutions and explain limitations related to solutions.

M1.8. Core Processes: Reasoning, problem solving, and communication
Students formalize the development of reasoning in Mathematics 1 as they use algebra, geometry, and statistics to make and defend generalizations. They justify their reasoning with accepted standards of mathematical evidence and proof, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

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M1.1. Core Content: Solving problems (Algebra)
Students learn to solve many new types of problems in Mathematics 1, and this first core content area highlights the types of problems students will be able to solve after they master the concepts and skills in this course. Throughout Mathematics 1, students spend considerable time with linear functions and are introduced to other types of functions, including exponential functions and functions defined piecewise. They learn that specific functions model situations described in word problems, and thus they learn the broader notion that functions are used to solve various types of problems. The ability to write an equation that represents a problem is an important mathematical skill in itself, and each new function provides students the tool to solve yet another class of problems. Many problems that initially appear to be very different from each other can actually be represented by identical equations. This is an important and unifying principle of algebra—that the same algebraic techniques can be applied to a wide variety of different situations.

M1.2. Core Content: Characteristics and behaviors of functions (Algebra)
Students formalize and deepen their understanding of functions, the defining characteristics and uses of functions, and the mathematical language used to describe functions. They learn that functions are often specified by an equation of the form $y = f(x)$, where any allowable x -value yields a unique y -value. Mathematics 1 has a particular focus on linear functions, equations, and systems of equations and on functions that can be defined piecewise, particularly step functions and functions that contain the absolute value of an expression. Students compare and contrast nonlinear functions, such as quadratic and exponential, with linear functions. They learn about the representations and basic transformations of these functions and the practical and mathematical limitations that must be considered when working with functions and when using functions to model situations.

M1.3 Core Content: Linear functions, equations, and relationships (Algebra, Geometry/Measurement, Data/Statistics/Probability)
Students understand that linear functions can be used to model situations involving a constant rate of change. They build on the work done in middle school to solve systems of linear equations and inequalities in two variables, learning to interpret the intersection of lines as the solution. While the focus is on solving equations, students also learn graphical and numerical methods for approximating solutions to equations. They use linear functions to analyze relationships, represent and model problems, and answer questions. These algebraic skills are applied in other Core Content areas across high school courses.

M1.4. Core Content: Proportionality, similarity, and geometric reasoning (Geometry/Measurement)
Students extend and formalize their knowledge of two-dimensional geometric figures and their properties, with a focus on properties of lines, angles, and triangles. They explain their reasoning using precise mathematical language and symbols. Students study basic properties of parallel and perpendicular lines, their respective slopes in the coordinate plane, and the properties of the angles formed when parallel lines are intersected by a transversal. They prove related theorems and apply them to solve problems that are purely mathematical and that arise in applied contexts. Students formalize their prior work with similarity and proportionality by making and proving conjectures about triangle similarity.

M1.5. Core Content: Data and distributions (Data/Statistics/Probability)
Students select mathematical models for data sets and use those models to represent, describe, and compare data sets. They analyze the linear relationship between two statistical variables and make and defend appropriate predictions, conjectures, and generalizations based on data. Students understand limitations of conclusions drawn from the results of a study or an experiment and recognize common misconceptions and misrepresentations.

M1.6. Core Content: Numbers, expressions, and operations (Numbers, Operations, Algebra)
Students see the number system extended to the real numbers represented by the number line. They

Mathematics 1 Performance Expectations

Solving Problems (algebra)

- M1.1.A Select and justify functions and equations to model and solve problems.
- M1.1.B Solve problems that can be represented by linear functions, equations, and inequalities.
- M1.1.C Solve problems that can be represented by a system of two linear equations or inequalities.
- M1.1.D Solve problems that can be represented by exponential functions and equations.

Characteristics and Behaviors of Functions (algebra)

- M1.2.A Determine whether a relationship is a function and identify the domain, range, roots, and independent and dependent variables.
- M1.2.B Represent a function with a symbolic expression, as a graph, in a table, and using words, and make connections among these representations.
- M1.2.C Evaluate $f(x)$ at a (i.e., $f(a)$) and solve for x in the equation $f(x) = b$.
- M1.2.D Plot points, sketch, and describe the graphs of functions of the form $f(x) = \frac{a}{x} + b$.

Linear Functions, Equations, and Relationships (algebra, geometry/measurement, data/statistics/probability)

- M1.3.A Write and solve linear equations and inequalities in one variable.
- M1.3.B Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships they represent.
- M1.3.C Identify and interpret the slope and intercepts of a linear function, including equations for parallel and perpendicular lines.
- M1.3.D Write and graph an equation for a line given the slope and the y -intercept, the slope and a point on the line, or two points on the line, and translate between forms of linear equations.
- M1.3.E Write and solve systems of two linear equations and inequalities in two variables.
- M1.3.F Find the equation of a linear function that best fits bivariate data that are linearly related, interpret the slope and y -intercept of the line, and use the equation to make predictions.
- M1.3.G Describe the correlation of data in scatterplots in terms of strong or weak and positive or negative.
- M1.3.H Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.

Proportionality, Similarity, and Geometric Reasoning (geometry/measurement)

- M1.4.A Distinguish between inductive and deductive reasoning.
- M1.4.B Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.
- M1.4.C Use deductive reasoning to prove that a valid geometric statement is true.
- M1.4.D Determine and prove triangle similarity.
- M1.4.E Know, prove, and apply theorems about parallel and perpendicular lines.
- M1.4.F Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.
- M1.4.G Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.

Data and Distributions (data/statistics/probability)

- M1.5.A Use and evaluate the accuracy of summary statistics to describe and compare data sets.
- M1.5.B Describe how linear transformations affect the center and spread of univariate data.
- M1.5.C Make valid inferences and draw conclusions based on data.

Numbers, Expressions, and Operations (numbers, operations, algebra)

- M1.6.A Know the relationship between real numbers and the number line, and compare and order real numbers with and without the number line.
- M1.6.B Determine whether approximations or exact values of real numbers are appropriate, depending on the context, and justify the selection.
- M1.6.C Recognize the multiple uses of variables, determine all possible values of variables that satisfy prescribed conditions, and evaluate algebraic expressions that involve variables.
- M1.6.D Solve an equation involving several variables by expressing one variable in terms of the others.

Additional Key Content (numbers, algebra)

- M1.7.A Sketch the graph for an exponential function of the form $y = ab^n$ where n is an integer, describe the effects that changes in the parameters a and b have on the graph, and answer questions that arise in situations modeled by exponential functions.
- M1.7.B Find and approximate solutions to exponential equations.
- M1.7.C Interpret and use integer exponents and square and cube roots, and apply the laws and properties of exponents to simplify and evaluate exponential expressions.
- M1.7.D Express arithmetic and geometric sequences in both explicit and recursive forms, translate between the two forms, explain how rate of change is represented in each form, and use the forms to find specific terms in the sequence.

Reasoning, Problem Solving, and Communication

- M1.8.A Analyze a problem situation and represent it mathematically.)
- M1.8.B Select and apply strategies to solve problems.
- M1.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- M1.8.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.
- M1.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- M1.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
- M1.8.G Synthesize information to draw conclusions, and evaluate the arguments and conclusions of others.
- M1.8.H Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.

arithmetic operations and properties and expand this understanding to include the symbolic language of algebra. Students understand the role of units in measurement, convert among units within and between different measurement systems as needed, and apply what they know to solve problems. They use derived measures such as those used for speed (e.g., feet per second) or determining automobile gas consumption (e.g., miles per gallon).

M2.6. Core Processes: Reasoning, problem solving, and communication

Students formalize the development of reasoning in Mathematics 2 as they use algebra, geometry, and probability to make and defend generalizations. They justify their reasoning with accepted standards of mathematical evidence and proof, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

**Mathematics content based on
Adopted Washington State
High School Mathematics Standards,
August 22, 2008,
Office of the Superintendent of Public
Instruction**

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Adopted Washington State Mathematics Standards

Mathematics 2 August 22, 2008

M2.1. Core Content: Modeling situations and solving problems (Algebra)

This first core content area highlights the types of problems students will be able to solve by the end of Mathematics 2. Students extend their ability to model situations and solve problems with additional functions and equations in this course. Additionally, they deepen their understanding and proficiency with functions they encountered in Mathematics 1 and use these functions to solve more complex problems. When presented with a word problem, students determine which function or equation models the problem and then use that information to write an equation to solve the problem. Turning word problems into equations that can be solved is a skill students hone throughout the course.

M2.2. Core Content: Quadratic functions, equations, and relationships (Algebra)

Students learn that exponential and quadratic functions can be used to model some situations where linear functions may not be the best model. They use graphical and numerical methods with exponential functions of the form $y = ab^x$ and quadratic functions to analyze relationships, represent and model problems, and answer questions. Students extend their algebraic skills and learn various methods of solving quadratic equations over real or complex numbers, including using the quadratic formula, factoring, graphing, and completing the square. They learn to translate between forms of quadratic equations, applying the vertex form to evaluate maximum and minimum values and find symmetry of the graph, and they learn to identify which form should be used in a particular situation. They interpret the meaning of problem solutions and explain their limitations. Students recognize common examples of situations that can be modeled by quadratic functions, such as maximizing area or the height of an object moving under the force of gravity. They compare the characteristics of quadratic functions to those of linear and exponential functions. The understanding of these particular types of functions, together with students' understanding of linear functions, provides students with a powerful set of tools to use

mathematical models to deal with problems and situations in advanced mathematics courses, in the workplace, and in everyday life.

M2.3. Core Content: Conjectures and proofs (Algebra, Geometry/Measurement)

Students extend their knowledge of two-dimensional geometric figures and their properties to include quadrilaterals and other polygons, with special emphasis on necessary and sufficient conditions for triangle congruence. They work with geometric constructions, using dynamic software as a tool for exploring geometric relationships and formulating conjectures and using compass-and-straightedge and paper-folding constructions as contexts in which students demonstrate their knowledge of geometric relationships. Students define the basic trigonometric ratios and use them to solve problems in a variety of applied situations. They formalize the reasoning skills they have developed in previous grades and solidify their understanding of what it means to mathematically prove a geometric statement. Students encounter the concept of formal proof built on definitions, axioms, and theorems. They use inductive reasoning to test conjectures about geometric relationships and use deductive reasoning to prove or disprove their conclusions. Students defend their reasoning using precise mathematical language and symbols. Finally, they apply their knowledge of linear functions to make and prove conjectures about geometric figures on the coordinate plane.

M2.4. Core Content: Probability (Data/Statistics/Probability)

Students formalize their study of probability, computing both combinations and permutations to calculate the likelihood of an outcome in uncertain circumstances and applying the binomial theorem to solve problems. They apply their understanding of probability to a wide range of practical situations, including those involving permutations and combinations. Understanding probability helps students become knowledgeable consumers who make sound decisions about high-risk games, financial issues, etc.

M2.5. Additional Key Content (Algebra, Measurement)

Students grow more proficient in their use of algebraic techniques as they use these techniques to write equivalent expressions in various forms. They build on their understanding of computation using

Mathematics 2 Performance Expectations

Modeling Situations and Solving Problems (algebra)

- M2.1.A Select and justify functions and equations to model and solve problems.
- M2.1.B Solve problems that can be represented by systems of equations and inequalities.
- M2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.
- M2.1.D Solve problems that can be represented by exponential functions and equations.
- M2.1.E Solve problems involving combinations and permutations.

Quadratic Functions, Equations, and Relationships (algebra)

- M2.2.A Represent a quadratic function with a symbolic expression, as a graph, in a table, and with a description, and make connections among the representations.
- M2.2.B Sketch the graph of a quadratic function, describe the effects that changes in the parameters have on the graph, and interpret the x -intercepts as solutions to a quadratic equation.
- M2.2.C Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.
- M2.2.D Solve quadratic equations that can be factored as $(ax + b)(cx + d)$ where a , b , c , and d are integers.
- M2.2.E Determine the number and nature of the roots of a quadratic function.
- M2.2.F Solve quadratic equations that have real roots by completing the square and by using the quadratic formula.
- M2.2.G Solve quadratic equations and inequalities, including equations with complex roots.
- M2.2.H Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.

Conjectures and Proofs (algebra, geometry/ measurement)

- M2.3.A Use deductive reasoning to prove that a valid geometric statement is true.
- M2.3.B Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.
- M2.3.C Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.
- M2.3.D Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.
- M2.3.E Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.
- M2.3.F Determine and prove triangle congruence and other properties of triangles.
- M2.3.G Know, prove, and apply the Pythagorean Theorem and its converse.
- M2.3.H Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.
- M2.3.I Use the properties of special right triangles (30° – 60° – 90° and 45° – 45° – 90°) to solve problems.
- M2.3.J Know, prove, and apply basic theorems about parallelograms.
- M2.3.K Know, prove, and apply theorems about properties of quadrilaterals and other polygons.
- M2.3.L Determine the coordinates of a point that is described geometrically.
- M2.3.M Verify and apply properties of triangles and quadrilaterals in the coordinate plane.

Probability (data/statistics/probability)

- M2.4.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).
- M2.4.B Given a finite sample space consisting of equally likely outcomes and containing events A and B , determine whether A and B are independent or dependent, and find the conditional probability of A given B .
- M2.4.C Compute permutations and combinations, and use the results to calculate probabilities.
- M2.4.D Apply the binomial theorem to solve problems involving probability.

Additional Key Content (algebra, measurement)

- M2.5.A Use algebraic properties to factor and combine like terms in polynomials.
- M2.5.B Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.
- M2.5.C Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.
- M2.5.D Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.

Reasoning, Problem Solving, and Communication

- M2.6.A Analyze a problem situation and represent it mathematically.
- M2.6.B Select and apply strategies to solve problems.
- M2.6.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- M2.6.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.
- M2.6.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- M2.6.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
- M2.6.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.
- M2.6.H Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.

M3.7. Additional Key Content (Geometry/Measurement)

Students formulate conjectures about circles. They use deductive reasoning to establish the truth of conjectures or to reject them on the basis of counterexamples. Students explain their reasoning using precise mathematical language and symbols. They apply their knowledge of geometric figures and their properties to solve a variety of both purely mathematical and applied problems.

M3.8. Core Processes: Reasoning, problem solving, and communication

Students formalize the development of reasoning in Mathematics 3 as they use algebra, geometry, and statistics to make and defend generalizations. They justify their reasoning with accepted standards of mathematical evidence and proof, using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

**Mathematics content based on
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August 22, 2008,
Office of the Superintendent of Public
Instruction**

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Adopted Washington State Mathematics Standards

Mathematics 3 August 22, 2008

M3.1. Core Content: Solving problems (Algebra)

The first core content area highlights the types of problems students will be able to solve by the end of Mathematics 3, as they extend their ability to solve problems with additional functions and equations. Additionally, they deepen their understanding of and skills related to functions they encountered in Mathematics 1 and 2, and they use these functions to solve more complex problems. When presented with a contextual problem, students identify a function or equation that models the problem and use that information to write an equation to solve the problem. For example, in addition to using graphs to approximate solutions to problems modeled by exponential functions, they use knowledge of logarithms to solve exponential equations. Turning word problems into equations that can be solved is a skill students hone throughout the course.

M3.2. Core Content: Transformations and functions (Algebra, Geometry/Measurement)

Students formalize their previous study of geometric transformations, focusing on the effect of such transformations on the attributes of geometric figures. They study techniques for using transformations to determine congruence and similarity. Students extend their study of transformations to include transformations of many types of functions, including quadratic and exponential functions.

M3.3. Core Content: Functions and modeling (Algebra)

Students extend their understanding of exponential functions from Mathematics 2 with an emphasis on inverse functions. This leads to a natural introduction of logarithms and logarithmic functions. They learn to use the basic properties of exponential and logarithmic functions, graphing both types of functions to analyze relationships, represent and model problems, and answer questions. Students apply these functions in many practical situations, such as applying exponential functions to determine compound interest and applying logarithmic functions to determine the pH of a liquid. In addition, students extend their study of functions to include

polynomials of higher degree and those containing radical expressions. They formalize and deepen their understanding of real-valued functions, their defining characteristics and uses, and the mathematical language used to describe them. They compare and contrast the types of functions they have studied and their basic transformations. Students learn the practical and mathematical limitations that must be considered when working with functions or when using functions to model situations.

M3.4. Core Content: Quantifying variability (Data/Statistics/Probability)

Students extend their use of statistics as they graph bivariate data and analyze its shape to make predictions. They calculate and interpret measures of variability, confidence intervals, and margins of error for population proportions. Dual goals underlie the content in the section: Students prepare for the further study of statistics and also become thoughtful consumers of data.

M3.5. Core Content: Three-dimensional geometry (Geometry/Measurement)

Students formulate conjectures about three-dimensional figures. They use deductive reasoning to establish the truth of conjectures or to reject them on the basis of counterexamples. They extend and formalize their work with perimeter, area, surface area, and volume of two- and three-dimensional figures, focusing on mathematical derivations of these formulas and their applications in complex problems. They use properties of geometry and measurement to solve both purely mathematical and applied problems. They also extend their knowledge of distance and angle measurements in a plane to measurements on a sphere.

M3.6. Core Content: Algebraic properties (Numbers, Algebra)

Students continue to use variables and expressions to solve both purely mathematical and applied problems, and they broaden their understanding of the real number system to include complex numbers. Students extend their use of algebraic techniques to include manipulations of expressions with rational exponents, operations on polynomials and rational expressions, and solving equations involving rational and radical expressions.

Mathematics 3 Performance Expectations

Solving Problems (algebra)

- M3.1.A Select and justify functions and equations to model and solve problems.
- M3.1.B Solve problems that can be represented by systems of equations and inequalities.
- M3.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.
- M3.1.D Solve problems that can be represented by exponential and logarithmic functions and equations.
- M3.1.E Solve problems that can be represented by inverse variations of the forms:

$$f(x) = \frac{a}{x} + b, \quad f(x) = \frac{a}{x^2} + b, \quad \text{and} \quad f(x) = \frac{a}{(bx+c)}.$$

Transformations and Functions (algebra, geometry/measurement)

- M3.2.A Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.
- M3.2.B Determine and apply properties of transformations.
- M3.2.C Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.
- M3.2.D Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.
- M3.2.E Construct new functions using the transformations $f(x-h)$, $f(x)+k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).

Functions and Modeling (algebra)

- M3.3.A Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.
- M3.3.B Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.
- M3.3.C Solve exponential and logarithmic equations.
- M3.3.D Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x-c} + d$, and solve related equations.
- M3.3.E Plot points, sketch, and describe the graphs of functions of the forms: $f(x) = \frac{a}{x} + b$, $f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx+c)}$, and solve related equations.
- M3.3.F Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.
- M3.3.G Solve systems of three equations with three variables.

Quantifying Variability (data/statistics/probability)

- M3.4.A Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.
- M3.4.B Calculate and interpret margin of error and confidence intervals for population proportions.

Three-Dimensional Geometry (geometry/measurement)

- M3.5.A Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.
- M3.5.B Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.
- M3.5.C Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.
- M3.5.D Apply formulas for surface area and volume of three-dimensional figures to solve problems.
- M3.5.E Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.
- M3.5.F Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.

Algebraic Properties (numbers, algebra)

- M3.6.A Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.
- M3.6.B Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.
- M3.6.C Add, subtract, multiply, and divide polynomials.
- M3.6.D Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.

Additional Key Content (geometry, measurement)

- M3.7.A Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.
- M3.7.B Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).
- M3.7.C Explain and perform constructions related to the circle.
- M3.7.D Derive and apply formulas for arc length and area of a sector of a circle.

Reasoning, Problem Solving, and Communication

- M3.8.A Analyze a problem situation and represent it mathematically.
- M3.8.B Select and apply strategies to solve problems.
- M3.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- M3.8.D Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.
- M3.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- M3.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.
- M3.8.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.
- M3.8.H Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.