Teaching and Learning Mathematics
. . . Best Practices . . .

MAJOR SHIFTS in the classroom environemnt need to move toward the following best practices:

- ★ All students having an equal opportunity to learn.
- ★ A balanced focus on conceptual understanding as well as on procedural fluency.
- ★ Active student engagement in problem solving, reasoning, communicating, making connections, and using multiple representations.
- ★ Students having the opportunity to explore, investigate, estimate, question, predict, and test ideas about math.
- ★ Students exploring and developing understanding for new mathematical concepts using materials they can touch and feel, either natural or manufactured.
- ★ Students discovering mathematics, not just doing many problems in a book.
- ★ The teacher guiding the students' learning, not dictating how it must be done.
- ★ Students having many opportunities to look at math in terms of daily life, connections within mathematics, and connections between mathematics and other subjects.
- ★ Technologically well-equipped learning centers in which technology is used to enhance understanding.
- ★ Incorporation of multiple assessments that are aligned with instructional goals and practices.
- ★ Mathematical authority that lies within the power of sound reasoning and mathematical integrity.

To achieve this vision, professional learning communities must support the work of teachers and must provide the time needed for professional collaboration, planning, and enhancement of teachers' knowledge of mathematics and pedagogy.

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Instructional Expectations & Strategies

ELEMENTS OF A GOOD LESSON — A comprehensive math program with a wealth of resources, will be of little use without the skill of teachers who are intentional in their practice and committed to making their classroom a mathematical community. The standards make it clear what the state expects students to learn and be able to do at each grade level. But, in an effort to meet these standards, what is expected of you, the teacher? What responsibilities do you have in relationship to the adopted mathematics program and to creating a math-friendly environment within your classroom? What should be happening in a good math lesson?

- A teaching practice which promotes student achievement must be one in which there is modeling by you. Students learn when they are involved in practice with you. While modeling a process or skill, you can check for understanding and adjust the pace and delivery as needed.
- A good math lesson is one in which questioning strategies and techniques support the goal of developing mathematical power for all students. Students are asked to justify and support their thinking. By evaluating "best" responses, students learn to solve problems in a variety of ways and use logical thinking.
- Throughout a good math lesson, the students are engaged. Students know the expectations, have participated in teacher-guided practice, and their hands and minds are focused on the learning that is taking place.
- In a good math lesson, students are purposefully using manipulatives. You have modeled their use, and the students are solving problems, conducting investigations, verifying results, and/or demonstrating understanding through hands-on learning.
- The math lesson should have a seamless progression from teacher modeling, to students working in pairs or groups, and, finally, to individual students working on their own.
- Students engaged in a good math lesson know how to work in small groups effectively. Their conversation, writing, and/or use of manipulatives involves each group member and is respectful of each student's contribution. They are able to accomplish the required task in the allotted time and feel comfortable reporting their results to the class. The grouping is flexible enough to accommodate varying abilities and the unique personalities of the students.
- You know the value of practice and individual assessments of learning. You have broken the lesson into small enough pieces and connected the concepts in such a way that student practice is an affirmation to the student of his/her understanding. The assignment provides enough flexibility to engage the most able learner, while addressing the needs of those for whom reinforcing the concept is most necessary.
- The good math lesson allows students to revise and improve their work or request extra help when needed. Students communicate their reflections of the lesson and what they have learned or still need to master. Students are able to assess their own learning and are comfortable using the resources available. They further know that progress is ongoing and are not discouraged or threatened when confusion occurs. For you, the lesson is never over.

In summary, the good math lesson is rich in mathematical investigations and represents an essential component of your balanced mathematics curriculum.

PUTTING ALL OF THE PIECES TOGETHER — We all know, however, that the teaching of mathematics, as does any other subject, reaches far beyond the boundaries of a single lesson, concept, or unit. For students to value mathematics and "make it their own," mathematics must be part of the environment of the classroom. Just as reading and writing have become integrated across the cur-

Instructional Expectations & Strategies … continued

riculum, so should mathematics be seen as integral to the teaching of all subjects. What might one expect to see in a math friendly environment?

- The classroom environment includes tools, games, hands-on materials, visuals, and other items available for activities and practice.
- Math vocabulary is used appropriately and consistently.
- Math skills are used in science, geography, and other subjects where appropriate and connections to math are actively pursued.
- Writing in mathematics helps reinforce the importance of clear communication skills in mathematics. Students are responsible for learning math vocabulary that has been introduced, experienced, and used as a part of daily mathematics, science, geography, health, etc.
- A variety of activities, observations, group and individual work is used to assess group progress and drive the pace and complexity of the curriculum.
- The teacher sets high expectations for the computational requirements of his/her grade level. The students are taught to use patterns and other shortcuts to help them be successful, and are provided many, and varied, opportunities for practice. Additionally, students are taught to recognize the appropriate use of mental math skills, paper and pencil calculations, and the calculator.
- Performance tasks and a variety of other assessments are used to assess individual progress and prescribe appropriate activities and practice for each student.
- The problem solving skills and strategies used for math power problems are integrated into the daily math program.
- Higher level questioning engages all students: during instruction, teacher modeling, and guided practice.

MATH POWER — Problem solving, reasoning logically, communicating understanding, and making connections, whether integrated into the daily lesson through the use of strategies and higher level questioning, or set apart in the problem of the day or WASL type questions, must be at the heart of every teacher's math curriculum. Many programs use a four-part problem solving guide as a model to help students understand the reasoning process they use when they solve problems. The four-step process, "understand, plan, solve, and look back," functions as a general guide for organizing methods, thinking, and strategies. It can help students reflect on what they are to do, what they have done, and what questions they need to ask and answer. By asking the guided questions and, when necessary, instructing directly, you can model this process as the class solves problems together. This interactive modeling helps student assimilate effective techniques as they take the ideas, approaches, and questions of proficient problem solvers and make them their own.

Lessons use interactive modeling to demonstrate and apply these problem solving strategies:

- Look for a pattern
- Make an organized list
- Make a table
- Guess and check
- Work backward
- Use logical reasoning
- Draw a diagram
- Solve a simplier problem
- Act it out
- Write an equation

Through investigations, students have a problem-based approach to mathematics that asks them to clarify their thinking and justify their results. The open-ended questions allow for growth of students' ability without the search for the one "right" answer.

Questioning Strategies & Techniques

The following questioning strategies and techniques support the goal of developing thinking for all students.

CLASSROOMS AS LEARNING COMMUNITIES —
Help students work together to make sense of learning, by asking and stimulating students to ask questions like the following:

"What do others think about what Janine said?"

"Does anyone have the same answer but a different way to explain it?"

"Would you ask the rest of the class that question?"

"Do you understand what they are saying?"

"Can you convince the rest of us that that makes sense?"

LOGIC AND EVIDENCE AS VERIFICATION OF UNDERSTANDING —
Help students to rely more on themselves to determine depth of understanding and whether something is correct by asking and stimulating students to ask questions like the following:

"Why do you think that?"

"Why is that true?"

"How did you reach that conclusion?"

"Does that make sense?"

"Can you make a visual representation to show that?"

REASONING —
Help students learn to reason by asking and stimulating students to ask questions like the following:

"Does that always work?"

"Is that true for all cases?"

"Can you think of a counter example?"

"How could you prove that?"

"What assumptions are you making?"

"How do you know?

CONJECTURING, INVENTING, AND PROBLEM SOLVING —
Help students learn to conjecture, invent, and solve problems by asking and stimulating students to ask questions like the following:

"What would happen if …? What if not?"

"Do you see a pattern?"

"What are some possibilities here?"

"Can you predict the next one? What about the last one?"

"What kind of predictions can you make?"

"How did you think about the problem?"

"What is alike and what is different about your method of solution and hers?"

CONNECTING LEARNING —
Help students to connect learning by asking and stimulating students to ask questions like the following:

"How does this relate to…?"

"What ideas that we have learned before were useful in solving this problem?"

"Have we ever solved a problem like this one before?"

"What did you find in the newspaper last night that reminds you of this?"

"Can you give me an example of …?"

"Could you explain this in another way?"

"Why does this matter?"

"What difference does it make?"

"How can you use this outside of school?"